MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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H. F. NEWALL, Esq., PRESIDENT, in the Chair.

John Bullock, M.A., 78 Airedale Avenue, Chiswick, W.;
Arthur Brunel Chatwood, B.Sc., Assoc.M.Inst.C.E., Astronomer to the Government of H.H. The Nizam, Hyderabad, Deccan, India;
F. R. Cripps, Esq., 22 Hornsey Rise Gardens, N.;
Henry Zouch Darrah, Esq., C.S.I., Allahabad, U.P., India;
Charles Gaskell Falkner, Esq., M.A., Ireton Bank, Rusholme, Manchester;
Harold Morris-Airey, Esq., M.Sc., Armstrong College, Newcastle-upon-Tyne; and
Henry William Moore, B.A., New University Club, St. James's Street, and 64 Curzon Street, Mayfair, W.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Hugh Cameron Campbell, Science Department, Surgeons' Hall, Edinburgh (proposed by Alex. D. Russell); and
Arthur Mackreth Deane, M.A., Canon of Chichester, Ferring Vicarage, Worthing, Sussex (proposed by F. J. W. Crowe).

The following were proposed by the Council as Associates of the Society:—

Benjamin Baillaud, Director of the Observatory, Paris;
C. V. L. Charlier, Director of the Observatory, Lund, Sweden;

E. B. Frost, Director of the Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.;
The Rev. J. G. Hagen, S.J., Director of the Vatican Observatory, Rome; and
Johannes Franz Hartmann, Astrophysical Observatory, Potsdam, Germany.

Seventy-one presents were announced as having been received since the last meeting, including, amongst others:—

J. G. Böhn, die Kunst-Uhren auf der K.-K. Sternwarte zu Prag, presented by the Prague Observatory; The History of the Geological Society of London, by H. B. Woodward, presented by the Society.

Sixteen charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich.

On the Lunar Inequalities due to the Motion of the Ecliptic and the Figure of the Earth. By Ernest W. Brown, Sc.D., F.R.S.

1. *The general disturbing function for the motion of the ecliptic.*—Let θ_1, θ_2, θ_3 be the angular velocities of a set of moving rectangular axes about themselves; x, y, z the co-ordinates; u, v, w the velocities of a particle with respect to these axes; and let F be the force function divided by the mass of the particle. Then the equations of motion are given by

$$\frac{du}{dt} - v\theta_3 + w\theta_2 = \frac{\partial F}{\partial x},$$

$$\frac{dv}{dt} - w\theta_1 + u\theta_3 = \frac{\partial F}{\partial y},$$

$$\frac{dw}{dt} - u\theta_2 + v\theta_1 = \frac{\partial F}{\partial z},$$

where

$$u = \frac{dx}{dt} - y\theta_3 + z\theta_2,$$

$$v = \frac{dy}{dt} - z\theta_1 + x\theta_3,$$

$$w = \frac{dz}{dt} - x\theta_2 + y\theta_1.$$

Put

$$H = \tfrac{1}{2}(u^2 + v^2 + w^2) - F - R,$$

where

$$R = vx\theta_3 - wx\theta_2 + wy\theta_1 - uy\theta_3 + uz\theta_2 - vz\theta_1;$$

then, if we assume that θ_1, θ_2, θ_3 are independent of x, y, z, u, v, w, we may write the equations of motion in the canonical form,

$$\frac{du}{dt} = -\frac{\partial H}{\partial x}, \quad \frac{dx}{dt} = \frac{\partial H}{\partial u},$$

$$\frac{dv}{dt} = -\frac{\partial H}{\partial y}, \quad \frac{dy}{dt} = \frac{\partial H}{\partial v},$$

$$\frac{dw}{dt} = -\frac{\partial H}{\partial z}, \quad \frac{dz}{dt} = \frac{\partial H}{\partial w}.$$

If R be neglected, the equations become of the same form as for fixed axes; hence R is the disturbing function for the motions of the axes. These motions are so small that we can neglect squares and higher products of the terms in R, and therefore can treat R as a disturbing function of the same nature as that used for ordinary planetary perturbations.

In order to obtain θ_1, θ_2, θ_3, we put i' for the inclination of the moving ecliptic (xy plane) to the fixed ecliptic (that of 1850·0), τ for the longitude of the node on the fixed ecliptic, L for the angle which the zx plane makes with the plane containing the poles of the fixed and moving ecliptics. Then, by Euler's equations,

$$\theta_1 = \frac{di'}{dt} \sin L - \sin i' \cos L \frac{d\tau}{dt},$$

$$\theta_2 = \frac{di'}{dt} \cos L + \sin i' \sin L \frac{d\tau}{dt},$$

$$\theta_3 = \frac{d\tau}{dt} \cos i' + \frac{dL}{dt}.$$

As L is at our disposal, we so take it that the distances of the origins of reckoning on the fixed and moving ecliptics from their common node are the same, that is, so that $L = 90^\circ - \tau$. The moving axis of x therefore passes through a "departure point."

When we are given the values of i', τ in terms of the time, we have the material necessary for the solution of the problem.

2. As a matter of fact, i', $\frac{di'}{dt}$, $\frac{d\tau}{dt}$ are so small that their squares and products may be neglected. We can therefore put

$$\sin i' = i', \quad \cos i' = 1, \quad \tau = \text{const.},$$

and obtain

$$\theta_1 = \frac{di'}{dt} \cos \tau, \quad \theta_2 = \frac{di'}{dt} \sin \tau, \quad \theta_3 = 0.$$

Whence

$$R = \frac{di'}{dt}\left\{(wy - vz)\cos\tau + (uz - wx)\sin\tau\right\},$$

or, since u differs from $\frac{dx}{dt}$ by a quantity of the same order as i', etc.,

$$R = \frac{di'}{dt}\left\{\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)\cos\tau - \left(x\frac{dz}{dt} - z\frac{dx}{dt}\right)\sin\tau\right\}$$

$$= \frac{di'}{dt}Q, \text{ suppose.}$$

The method of the variation of arbitrary constants permits us to substitute in Q the values of x, y, z, and of their derivatives in terms of the arbitrary constants and the time as found in the solution of the main problem of the lunar theory. These values can be obtained directly from the results which I have given in my papers in the *Memoirs* of the Society on this subject.

3. The value of i' is of the form

$$i' = pt + P,$$

where p is a constant and P a sum of periodic terms whose arguments depend on those of the motions of the Earth and planets. The latter terms have generally been neglected, but I shall show that they give rise to a few terms which are not insensible. At this stage the question arises as to what moving ecliptic we shall refer the motion of the Moon. We may refer it to the *actual* ecliptic, in which case I have found by calculation that P gives rise to a number of primary terms of short period and a few of long period. Or we may refer it to the *mean* ecliptic, *i.e.* put $P = 0$ in the above formulæ; but if we do so, it will be necessary to introduce the latitude of the Sun above this mean ecliptic into F. With this latter method, I have found that all the primary short-period terms practically disappear and the long-period terms have rather smaller coefficients than with the former plane of reference.

I therefore adopt the mean ecliptic together with the one or two very minute terms which are of long period relative to that of the Moon's node, and which do not then give rise to any terms in the Moon's co-ordinates. Hence, for the computation, we may put

$$R_1 = pQ.$$

4. The principal part of the force function for the Moon's motion under the influence of the Earth and Sun is

$$F = \frac{\mu}{r} + \frac{m'}{r'^3}\left\{\frac{3}{2}\frac{(xx' + yy' + zz')^2}{r'^2} - \frac{1}{2}(x^2 + y^2 + z^2)\right\}$$

where x', y', z' are the co-ordinates of the Sun.

The main problem is solved with $z' = 0$. Hence the additional portion due to z' is, if we neglect squares of z',

$$R_2 = \frac{m'}{r'^3} \cdot \frac{3(xx' + yy')zz'}{r'^2}.$$

With the notation and limitations of § 2, 3 we have

$$z' = i''(y' \cos \tau - x' \sin \tau).$$

Also, with this value for z' and by means of the equations $\frac{d^2x}{dt^2} = \frac{\partial F}{\partial x}$, etc., it is easy to show that

$$\frac{m'}{r'^3} \frac{3(xx' + yy')zz'}{r'^2} = i'' \left\{ \left(z\frac{d^2y}{dt^2} - y\frac{d^2z}{dt^2} \right) \cos \tau - \left(z\frac{d^2x}{dt^2} - x\frac{d^2z}{dt^2} \right) \sin \tau \right\}.$$

Hence, since τ is a constant, we obtain

$$R_2 = -i''\frac{dQ}{dt},$$

where for i'' we substitute its periodic portions, omitting those mentioned at the end of § 3.

5. It is of some interest to compare the two disturbing functions R_1, R_2. In general we have

$$R_1 = Q\frac{di''}{dt}, \quad R_2 = -i''\frac{dQ}{dt} = Q\frac{di''}{dt} - \frac{d}{dt}(i''Q).$$

Thus the effect of the rotation of the axes alone, apart from the motion of the Sun out of the fixed plane, is to introduce a term $\frac{d}{dt}(i''Q)$. From these expressions we may also prove the statements made in § 3 with reference to the presence of long- and short-period terms, remembering the two forms which have been given for $\frac{dQ}{dt}$. In fact, the terms of long period in Q are relatively much larger than those in $\frac{dQ}{dt}$ owing to the presence of small divisors; for a term of *very* long period in R_2, the portion arising from $\frac{d}{dt}(i''Q)$ is small compared with that arising from R_1.

6. The computation from the two functions R_1, R_2, as defined in §§ 3, 4, give the results which follow. The values of Leverrier * have been used for i', τ, and the method is the same as that which I have used for planetary inequalities in general.

I put w_1, w_2, w_3 for the mean longitudes of the Moon, of its perigee and of its node; l, D, F, n, e, γ are the quantities as defined by Delaunay; T, V, J are the mean longitudes of the

* *Ann. Obs. Paris* (*Mém.*), vol. iv. pp. 13–21.

Earth, Venus, and Jupiter, respectively; $\phi = w_3 + 96^\circ{\cdot}2$; $\phi' = w_1 + 96^\circ{\cdot}2$.

Then the variations of the elements for the principal inequality, argument ϕ, arising from R_1, are

$$\begin{aligned} \delta w_1 &= -\ 0''{\cdot}289 \sin\phi, & \delta n &= +0''{\cdot}0014 n \cos\phi, \\ \delta w_2 &= +\ 0''{\cdot}840 \sin\phi, & \delta e &= \quad\ {\cdot}000 \cos\phi, \\ \delta w_3 &= -15''{\cdot}59 \sin\phi, & \delta\gamma &= +0''{\cdot}698 \cos\phi. \end{aligned}$$

These agree very closely with the values found by Professor Newcomb.* They differ a little more from those found by Dr. Hill;† he, however, used a method with a literal development, and the small differences are probably due to slow convergence.

The principal term in latitude, argument ϕ', has a coefficient of $+1''{\cdot}382$.

7. The new terms arising from R_2 are, if we omit terms with coefficients less than $0''{\cdot}010$,

In Longitude.	In Latitude.
$+0''{\cdot}019 \sin(\phi + 5T - 3V + 119^\circ)$	$+0''{\cdot}077 \sin(\phi' + 5T - 3V + 119^\circ{\cdot}4)$
$+0''{\cdot}003 \sin(\phi + 2w_2 - 2J + 90^\circ)$,	$+0''{\cdot}030 \sin(\phi' - 5T + 3V - 119^\circ{\cdot}4)$
	$+0''{\cdot}035 \sin(\phi' + 2J + 72^\circ)$
	$+0''{\cdot}018 \sin(\phi' - 2J - 72^\circ)$.

The two terms in longitude are "primary" terms of long period. The second one is of interest, since its period is that of the empirical term which appears to represent best the long-period difference between theory and observation, namely, about 280 years. Its coefficient is, however, too small to account for this difference.‡ There is a term with the same argument due to the direct effect of the planets, but its coefficient is also insensible. The terms in latitude are "secondary," and have periods approximating to a month.

8. *Inequalities arising from the figure of the Earth.*—These have been computed very fully by Dr. Hill,§ who used the method of Delaunay, and there would be no reason for the mention of them here if it were not that slight doubts have arisen concerning their degrees of accuracy on account of slow convergence. I have used the equations of variations and the results of my lunar theory which allow the coefficients to be obtained easily within $0''{\cdot}003$. This method is brief, and the computations occupied about eight

* Carnegie Institute, publ. 72, p. 132. The difference in the value of δn is due to a difference in the definition of this variation.

† *Annals of Math.*, vol. i. p. 57. Coll. Works, vol. ii. p. 77.

‡ In a note on p. 170 of the present volume, and in this paper as read before the Society, I gave $0''{\cdot}21$ as the value of this coefficient. A recalculation with the first form of R_2 revealed an error in the former computation.

§ *Amer. Eph. Papers*, vol. iii. pp. 201–344. Coll. Works, vol. ii. pp. 181–320.

days. I put $\psi = w_3 +$ the precession, and obtain for the variations of the elements

$$\begin{aligned} \delta w_1 &= +\ 7''\cdot317 \sin\psi\,, & \delta n &= -\ 0''\cdot009 \cos\psi\,, \\ \delta w_2 &= -\ 2''\cdot092 \sin\psi\,, & \delta e &= +\ 0''\cdot002 \cos\psi\,, \\ \delta w_3 &= +\ 96''\cdot69\ \sin\psi\,, & \delta\gamma &= -\ 4''\cdot351 \cos\psi\,. \end{aligned}$$

The short-period terms add $-\ 0''\cdot017 \sin(\psi + \mathrm{F})$ to the latitude. The principal term in longitude is δw_1, and the principal elliptic terms are $+\ 0''\cdot519 \sin(\psi + l) + 0''\cdot515 \sin(\psi - l)$. The principal term in latitude is $-\ 8''\cdot355 \sin(\psi + \mathrm{F})$, and that with argument $\psi - \mathrm{F}$ is $+\ 0''\cdot338 \sin(\psi - \mathrm{F})$. When Hill's results are reduced to the value of the ellipticity used above (1/296·3),* my coefficient of the principal term in latitude agrees with his, and my coefficient of the principal term in longitude is 0″·030 less. The coefficients for the term with argument $\psi - \mathrm{F}$ also agree; this fact furnishes a useful test, since this coefficient is the difference of two numbers each nearly thirteen times as large as the coefficient. The two principal elliptic terms are 0″·021, 0″·017, respectively, greater than those of Hill. An examination of Hill's literal developments for the coefficients shows that the small differences can all be explained by slow convergence.

The complete results for the classes of terms considered here will be given in the fifth (concluding) part of my *Theory of the Motion of the Moon*, with the terms due to the action of the planets and to perturbations not considered in the previous parts.

New Haven, Conn.:
1908 *February* 1.

Postscript (1908 *April* 4).—The terms of the second order have been examined and produce nothing sensible. However, in consequence of my attention being called by Dr. Hill to a doubt as to whether there was a portion of the secular acceleration due to the figure of the Earth terms being referred to a moving ecliptic, I made an actual computation of this second order perturbation and found that the greatest effect could be exhibited in the form of a term of period about 15,000 years and coefficient 0″·15. This, equivalent to a secular acceleration of 0″·0001 within historic times, is entirely insensible. The computation will be given in chapter xiv. of my memoir just mentioned.

* Corresponding to the result marked (β) in *M.N.*, vol. lxiv. p. 531.

Note on the newly discovered Eighth Satellite of Jupiter, photographed at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Since the date of the last communication successful photographs have been obtained of this object on the following nights:—March 27, 31, and April 3. From these it would appear that the object discovered is a satellite of Jupiter very much more distant from the planet than the sixth or seventh satellites. The following are the places deduced from photographs taken with the 30-inch reflector.

Date and G.M.T.			Exposure.	Apparent R.A.			Apparent Dec.			Sat. VIII—Jupiter. R.A.		Dec.	
1908.	d	h m	min.	h	m	s	°	′	″	′	″	′	″
Jan.	27	12 41·4	64	8	45	51·86	+18	5	1·4	−0	8·29	−43	17·3
Feb.	1	11 52·1	111		43	20·67		17	36·3	+0	2·20	41	41·1
	3	10 27·3	80		42	21·90	18	22	25·7	0	6·37	41	4·2
	22	10 56·6	100		33	35·41	19	5	49·8	0	48·38	33	58·1
	23	8 33·8	42		33	13·98		7	37·6	0	50·31	33	35·7
	24	12 24·9	80		32	47·37		9	51·3	0	53·15	33	9·2
	27	10 59·4	70		31	42·89		15	18·1	0	59·94	31	59·7
	28	11 28·6	80		31	21·76		17	6·7	1	2·34	31	35·2
Mar.	27	8 54·3	43		26	32·33		47	53·5	2	11·35	20	59·9
	31	9 16·5	165		26	40·24		49	12·3	2	21·55	19	38 1
Apr.	3	9 51·7	90	8	26	54·38	+19	49	38·8	+2	28·99	−18	40·2

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

These places are definitive, replacing those given in the previous number, p. 373, which are provisional only.

The new satellite has also been photographed by Dr. Max Wolf at Heidelberg on March 3, 23, and 24, and at the Lick Observatory on March 8.

[A photograph of Jupiter's Eighth Satellite, as well as of the Sixth and Seventh Satellites, has since been obtained on April 24.]

Note by Mr. Melotte.

Throughout this and previous oppositions, in examining the photographs of Jupiter's sixth and seventh satellites a good look-out has been kept, with a view to detecting other satellites, should any exist. In view of the large gap between the orbits of the inner satellites and the sixth and seventh, the existence of others would appear not unlikely. On several occasions previous to this, suspected images had been noticed on the plates, and in some

cases (notably on four plates in 1905) images had been found on plates taken a few days before or after, which might possibly correspond. But the difficulty of distinguishing between photographic defects and true images is naturally very considerable, and, until the present occasion, it had never been possible in any case to obtain sufficient evidence to decide whether the images were real. A record of the positions of all suspected images is made for future reference.

Royal Observatory, Greenwich:
1908 *April* 10.

First approximation to the orbit of J VIII = CJ.

By A. C. D. Crommelin, B.A.

The hypothesis of retrograde motion gives a much more reasonable value of the distance of this object from Jupiter than the hypothesis of direct motion; it has therefore been provisionally adopted. Assuming that the distance of the object from Jupiter has remained sensibly constant during the period of observation (this assumption being justified by the uniformity of the rate of motion, when corrected for the varying distance and direction of the planet from the Earth), it is possible to deduce this distance by the curvature of the apparent path (the greater part of this curvature being due to the Earth's motion).

The result of several successive approximations gives the following values:—

Distance from Jupiter at inferior geocentric conjunction, in astronomical units.	0·213
Date of this conjunction . . .	1908 Feb. 20·0
R.A. of N. pole of orbit plane . .	336° 25′
N. dec. „ „ „ . .	54 48
Inclination to Jupiter's orbit . . .	32°
Daily angular motion about Jupiter .	0°·288

This satisfies all the observations within some 20″; a closer approximation could doubtless be obtained, but it is hardly worth while to do so till enough of the orbit has been described to make a reliable estimate of the solar perturbations, which must be extremely large. The rate of linear motion appears to have diminished by 1 per cent. in the period March 3 – April 3 as compared with Jan. 27 – March 3. This would be sufficiently explained by the Variation causing the motion to be most rapid at New and Full Moon.

The daily angular motion in an undisturbed orbit at distance 0·213 is 0°·310, with which the value 0°·288 is in satisfactory accord,

especially as the effect of the Variation both on the distance and the velocity must be considerable. The sidereal period is probably between three and four years, the synodic period being about one year shorter if the motion be retrograde.

The mean distance of satellite VII from Jupiter is 0·0785, so that the value of the distance found for the new body is about three times as great.

Table giving approximate values of the perturbations of Halley's Comet by Jupiter and Saturn in the first and fourth quadrants of the orbit. By P. H. Cowell, M.A., F.R.S., and A. C. D. Crommelin, B.A.

A sufficient number of revolutions of the comet have now been computed to enable the perturbations to be tabulated as functions of g', the mean anomaly of the disturbing planet at the adjacent perihelion passage. The formula for determining g' was given in *M.N.*, lxviii. 3, p. 177. The table is not intended to supersede calculation, but it serves—

(1) To form a good first approximation to the date of any perihelion passage, as a preliminary to more exact calculation.

(2) To detect any serious errors of calculation.

(3) To readily correct computations made with an erroneous assumed date of perihelion, by means of the rate of change of the tabular quantities.

	Jupiter, 1st quad.			Jupiter, 4th quad.			Saturn, 1st quad.			Saturn, 4th quad.	
g'	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$
°	″	″	″	″	″	″	″	″	″	″	″
0	+·85	−210	+140	+·26	−460	− 60	+·19	0	+ 20	−·03	+ 10
10	·93	130	160	·32	450	90	·14	− 30	20	·06	0
20	·99	− 30	180	·37	430	120	·09	50	20	·10	0
30	1·05	+100	200	·43	410	140	·07	60	20	·15	+ 10
40	1·10	220	200	·48	380	150	·05	50	10	·19	20
50	1·13	340	190	·54	350	160	·04	30	10	·23	30
60	1·14	450	170	·59	320	170	·03	20	10	·24	20
70	1·13	560	140	·64	270	170	·02	− 10	10	·23	+ 10
80	1·10	660	90	·67	200	160	·01	+ 10	10	·21	− 10
90	·97	670	40	+·38	−100	130	+·01	20	+ 10	·18	30
100	·71	610	+ 10	−·27	+130	110	·00	40	0	·15	50
110	·39	470	− 30	·90	420	70	−·01	50	0	·12	60
120	+·03	330	70	1·20	620	− 30	·01	60	0	·10	70
130	−·18	210	110	1·28	710	+ 10	·02	60	0	·07	80
140	·24	100	120	1·25	640	50	·03	60	0	·05	80

	Jupiter, 1st quad.			Jupiter, 4th quad.			Saturn, 1st quad.			Saturn, 4th quad.		
g'	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$	$\int dn$	$\int d\varpi$	$\int d\zeta$
°	″	″	″	″	″	″	″	″	″	″	″	″
50	·27	+ 20	130	1·20	560	90	·04	50	− 10	·03	90	0
60	·27	− 50	130	1·16	470	120	·04	40	10	− ·02	80	0
70	·27	120	130	1·12	370	140	·05	30	10	·00	80	− 10
80	·27	170	130	1·08	270	160	·06	20	20	+ ·01	80	20
90	·26	220	120	1·03	180	170	·06	+ 10	20	·03	70	30
00	·24	280	110	·97	+ 70	180	·06	− 10	10	·04	50	20
10	·23	340	90	·90	− 30	190	·06	20	10	·05	40	20
20	·21	390	80	·83	130	200	·05	40	− 10	·06	20	10
30	·19	460	60	·76	230	200	·04	50	0	·06	− 10	10
40	·16	520	40	·68	320	190	·03	70	0	·06	+ 10	10
50	·12	580	− 20	·58	420	180	− ·02	80	0	·06	20	10
60	− ·06	620	0	·48	510	160	·00	90	+ 10	·06	30	10
70	·00	640	+ 20	·38	550	130	+ ·02	90	20	·05	40	10
80	+ ·07	640	40	·29	570	110	·04	100	20	·04	50	10
90	·16	630	60	·19	590	90	·07	90	30	·03	50	10
00	·25	610	80	·11	600	70	·09	90	30	·02	60	− 10
10	·34	580	90	− ·04	600	40	·12	90	30	+ ·01	60	0
20	·44	530	100	+ ·03	580	+ 10	·15	80	30	·00	50	0
30	·54	470	110	·09	560	− 10	·19	60	30	·00	30	0
40	·65	400	120	·14	540	30	·22	40	30	− ·01	+ 10	0
50	+ ·75	− 310	+ 130	+ ·20	− 510	− 40	+ ·22	− 20	+ 30	− ·02	0	+ 10

To $\int d\zeta$ in the first quadrant must be added the corresponding value of $\int dn \times$ period in days.

The argument g' is the value of the planet's mean anomaly at the preceding perihelion passage for the first quadrant, at the following passage for the fourth quadrant.

Note on the condition for the passage of the Earth through the plane of Saturn's Ring. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The interesting observations made recently on the ring seen edgewise have brought several inquiries as to the recurrence of this beautiful phenomenon; and the following note, originally made some years ago in consequence of an inquiry from Mr. C. T. Whitmell, may be of use to others. The late Mr. R. A. Proctor gives in his book *Saturn and his System* a general account of the manner in which the Earth may pass through the plane of the ring, either once or three times, at each favourable opportunity. The present note gives the explanation in more compact form.

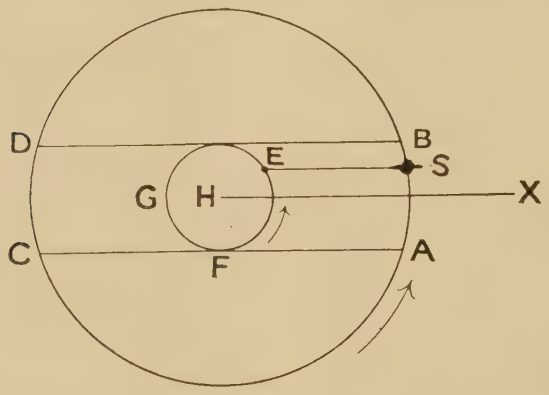


FIG. 1.

2. Let H be the Sun (fig. 1), EGF, ABDC, the orbits of the Earth and Saturn, supposed circular and in one plane. Let BD and AC be two tangents to the Earth's orbit parallel to the plane of Saturn's ring. Then if S be Saturn and E the Earth at a time of passage through the ring, ES must be parallel to BD or AC. Hence Saturn must be either in the portion AB or DC of his orbit. Since his orbit is ten times the size of the Earth's, these favourable opportunities are confined to limited periods which recur at long intervals. AB is about $\frac{1}{15}$ of the semi-orbit, and is described in about a year, so that the Earth meanwhile makes a complete revolution. If, when Saturn is near A, the Earth is near F, then there may be three passages through the ring.

3. To find the condition in exact terms, take HX parallel to AC or BD as axis of x. Let the radius of the Earth's orbit be unity, and that of Saturn's orbit n^2. The co-ordinates of the Earth may be written

$$x_1 = \cos t,\ y_1 = \sin t,$$

and of Saturn (remembering Kepler's Third Law)

$$x_2 = n^2 \cos.(t-\alpha)/n^3,\ y_2 = n^2 \sin(t-\alpha)/n^3,$$

where the unit of time is $1/2\pi$ of a year and the origin of time is taken when the Earth lies on HX, Saturn then being an angle α/n^3 behind it.

The condition for passage of the Earth through the ring is thus

$$y_1 = y_2 \quad \text{or} \quad \sin t = n^2 \sin(t-\alpha)/n^3,$$

a transcendental equation, which we can only solve by approximation.

4. Let us draw the curves

$$\text{(a)} \quad y = \sin t,$$
$$\text{(b)} \quad y = n^2 \sin(t-\alpha)/n^3,$$

then the required values of t will be given by their intersections. Now the curves are both sine curves, differing only in period and amplitude.

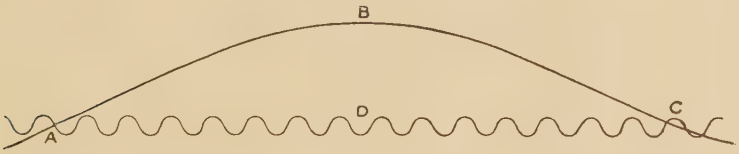


FIG. 2.

The Earth's curve (a) alternates much more rapidly, as in ADC (fig. 2). Saturn's curve (b) is a broad sweep, ABC, and we see at once how the crossings of the two curves correspond to the favourable opportunities of fig. 1; and further, that there may be a single intersection as at A, or a triple intersection as at C. There must always be an odd number of intersections; whether there can be five, seven, or more depends on the actual dimensions of the curves. If there is only one intersection, notice that the slopes of the two curves are in opposite directions, *i.e.* the planets are on opposite sides of the Sun; so that Saturn will be near conjunction with the Sun, and the phenomenon will not be so readily observable. If there are three intersections, the middle one is near opposition and the other two near quadratures.

5. In fig. 3, let ABCDE be two consecutive waves of the Earth's curve, and let FG, HK, LM represent three possible positions of a portion of Saturn's curve, each touching the Earth's curve in one point (F, K, L) and cutting it in another (G, H, M). It is clear that if Saturn's curve cuts the axis OX between T and V there will be only one intersection; if between V and W there will be three. The separating cases are thus defined by the three values of α, $\mathrm{OT} = \alpha_1$, $\mathrm{OV} = \alpha_2$, $\mathrm{OW} = \alpha_3$, and it is easily seen that $\alpha_2 = 2\pi - \alpha_1$, $\alpha_3 = \alpha_1 + 2\pi$.

6. To find the value of α_1, we have the conditions that both y and $\frac{dy}{dt}$ must be the same for both curves.

$$\left.\begin{aligned} \therefore \quad \sin t &= n^2 \sin (t-\alpha)/n^3 \\ n^3 \cos t &= n^2 \cos (t-\alpha)/n^3 \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

$$\therefore \quad \sin^2 t + n^6 \cos^2 t = n^4 . \quad . \quad . \quad . \quad (3)$$

$$n^6 \sin^2 (t-\alpha)/n^3 + \cos^2 (t-\alpha)/n^3 = n^2 \quad . \quad . \quad (4)$$

whence $$\cos^2 t = (n^4 - 1)/(n^6 - 1)$$

so that $$\cos t = \frac{1}{n}\left\{1 + \frac{1}{n^4+n^2}\right\}^{-\frac{1}{2}} \quad . \quad . \quad (5)$$

and further $$\sin (t-\alpha)/n^3 = \frac{1}{n^2}\left\{1 + \frac{n^2+1}{n^4}\right\}^{-\frac{1}{2}} \quad . \quad . \quad (6)$$

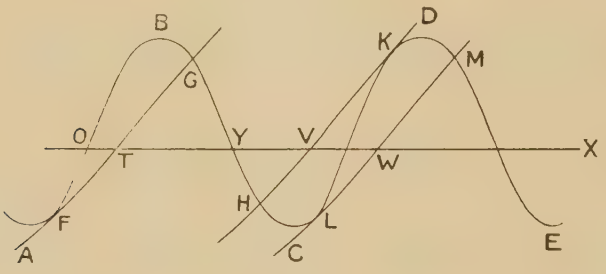


FIG. 3.

Since n^3 is 29·46, the number of years in which Saturn revolves round the Sun, $n = 3{\cdot}088$, $n^2 = 9{\cdot}54$, $n^4 = 91{\cdot}0$. Neglecting quantities of the order n^{-4} in t,

$$t = -\frac{\pi}{2} + \frac{1}{n} + \frac{1}{6n^3}, \quad (t-\alpha) = -n + \frac{1}{2n} - \frac{1}{24n^3}$$

$$\therefore \quad \alpha = -\frac{\pi}{2} + n + \frac{1}{2n} - \frac{5}{24n^3} \quad . \quad . \quad . \quad (7)$$

Thus for Saturn-Earth $\alpha = 1{\cdot}683$, or 96° 25′. Or we can, of course, obtain the numerical values of t and $(t-\alpha)/n^3$ directly from equations (5) and (6).

7. Hence $$\alpha_1 = 96^\circ, \quad \alpha_2 = 264^\circ, \quad \alpha_3 = 456^\circ.$$

And in the long-run, since the period of Saturn is incommensurable with that of the Earth, the values of α will be distributed uniformly, so that in a long series of years the chance of three intersections is

$$(\alpha_3 - \alpha_2)/360^\circ = 192/360 = {\cdot}53,$$

and the chance of one intersection is

$$(\alpha_2 - \alpha_1)/360^\circ = 168/360 = {\cdot}47.$$

Starting from any crossing with a value α_0 for α, the next crossing *in the same direction* will be at a point

$$\frac{t-\alpha_0}{n^3} = 2\pi$$

or $$t = \alpha_0 + 29{\cdot}46 \times 2\pi$$

$$= \alpha_0 + 166^\circ \text{ omitting } 29 \times 2\pi.$$

The intermediate crossing is in the opposite direction and at the opposite point of the orbit. The eccentricity of Saturn's orbit is so large that the intermediate value of α cannot be inferred by elementary considerations. There will be practically two separate series of values of α referring to the two nodes, each increasing by the difference 166°. But this value 166° is only approximate, and is modified by the motion of the ring plane. It appears from the subjoined table that at present the average value is about 162°.

8. The dates when Saturn crosses the line of nodes, or more properly when his ring-plane passes through the Sun, are given by Mr. Proctor on p. 223 (Table X.) of his book above mentioned (*Saturn and his System*). It will suffice here to give a few of them to the nearest day :—

Year.	Sun in plane of Ring.	Earth's Passage.	Fractions of Year. Sat.	Earth.	Diffce.		α.	
1789	Oct. 7	Sept. 13	·764	·698	·066	...	24°	...
1803	June 23	Mar. 12	·473	·192	...	·281	...	101°
1819	Mar. 23	Sept. 14	·221	·700	·521	...	187	...
1832	Dec. 4	Mar. 12	·923	·192	...	·731	...	263
1848	Sept. 3	Sept. 14	·671	·700	·971	...	349	...
1862	May 18	Mar. 12	·375	·192	...	·183	...	66
1878	Feb. 14	Sept. 15	·120	·703	·417	...	150	...
1891	Oct. 29	Mar. 13	·824	·195	...	·629	...	226
1907	July 29	Sept. 16	·572	·706	·866	...	312	...

Columns 1 and 2 above are from Mr. Proctor's book; column 3 is filled in from the *Nautical Almanac* by finding the date when the Earth had the same heliocentric longitude as Saturn had on the date given in the column before. The next two columns give these dates in fractions of a year, and the differences are the values of α in the unit adopted in § 3, viz. $1/2\pi$ of a year, more conveniently expressed in degrees in the next columns. The two series are kept separate, and it will be seen that the common difference is about 162° or 163°. If it were exactly $162^\circ = 180^\circ - 18^\circ$, the numbers would repeat after twenty terms, and we should have the series as follows :—

First Series.

Year	1789	1819	1848	1878	1907	1937	1966	1996	2025	2055	2084
α	25°	187°	349°	151°	313°	115°	277°	79°	241°	43°	205° etc.
No intersect^{ns}.	3	1	3	1	3	1	3	3	1	3	1

Second Series.

Year	1803	1832	1862	1891	1921	1950	1980	2009
α	101°	263°	65°	227°	29°	191°	83°	245° etc.
No intersect^{ns}.	1	3	3	1	3	1	3	1

Under the actual conditions, the series will slowly diverge from these, just as the cycle of total solar eclipses on the Earth slowly changes.

9. If the slope of Saturn's curve were less relatively to that of the Earth, there might be five intersections. For observers on Venus or Mercury this might be the case. The limiting case when five intersections are just possible is when a Saturn curve through the point Y (fig. 3), just touches both the adjacent curves near F and K, and then equations (2) are satisfied by $\alpha = \pi$.

$$\therefore \text{ from equation (7)} \quad n = \frac{3\pi}{2} - \frac{1}{2n} - \frac{5}{24n^3} = 4{\cdot}54 \text{ approx.}$$

For values of n greater than 4·54 there will accordingly be three or five intersections, and five will occur with greater frequency as n increases.

Now Bode's Law gives for the successive relative values of n^2

4	7	10	100	[388]
Mer.	Ven.	Earth.	Saturn.	Neptune.

Thus for Saturn-Venus, $n^2 = 14{\cdot}3$, $n = 3.8$

for Saturn-Mercury, $n^2 = 25$, $n = 5{\cdot}0$

Hence Saturn's ring can disappear five times to Mercury, but not to Venus. The value of n for Saturn-Neptune is too small for any but three intersections. It will readily be seen that the above procedure is applicable to the outer planets, the curves interchanging character.

10. Although Saturn's ring is a special problem, there are cases somewhat similar, *e.g.* the changes in declination of another planet. Times when the declination vanishes are times when the planet passes through the Earth's equator, and thus would be in the plane of its ring if it had one. The three intersections can readily be traced in the tabulated declinations of (say) Venus.

The Variability of the Nucleus of the Planetary Nebula N.G.C. 7662. 1860·0, α $23^h\ 19^m\ 11^s$, $\delta + 41°\ 46'·0$.

By E. E. Barnard. (Plate 12.)

This remarkable nebula (N.G.C. 7662) is situated in Andromeda. In an ordinary telescope it looks like a small round disc of a bluish-green colour. In the 4-inch finder of the 40-inch telescope it is only distinguishable from an ordinary star of the 8^m or $8\frac{1}{2}^m$ by a slight fuzziness of the image. In the great telescope, however, under good conditions, it is a beautiful object —a slightly elliptical disc with quite sharply defined outlines. Unsymmetrically placed on this is a roughly elliptical broken ring of greater brightness. The interior of this ring is dark, but not black, and in this, approximately central, is ordinarily a faint stellar nucleus. The broken ring (broken on its n.p. side, and with a smaller break opposite this) is of a bright bluish-green colour, while the general disc of the nebula often has the appearance of a warm or faintly reddish tint. The northern part of this inner ring is the brightest part of the nebula. The nucleus, with which I am going to deal specially, is sometimes of a bright yellowish colour, though ordinarily it is faint and apparently colourless. There is a faint star of the 15·5 or 16 magnitude exactly on the following edge of the outer disc.

Shortly after beginning to observe this nebula in 1897 I suspected that the nucleus was variable (see *Astrophysical Journal*, xiv. p. 153). Familiarity with the changing aspect of a star involved in nebulosity, due to moonlight, bad seeing, etc., has made me extremely cautious in this matter. I have, however, for several years, established conclusively the fact that the nucleus of 7662 is actually variable to an extent of upwards of three magnitudes. At times it has appeared as a bright yellowish star of about the 12th magnitude; and at other times, equally favourable, it has been either entirely invisible or excessively faint. Though I have not definitely determined the period, the light changes seem to cover about 28 days. The light curve is still undetermined; the normal condition, however, seems to be faint—the nucleus remaining bright for a few days only.

I have collected here the principal observations of the brightness of the nucleus, etc., obtained with the 40-inch telescope in the past ten years.

On 1904 August 6, among other objects, I showed this nebula to Professor H. H. Turner. Though we both looked very carefully, under fairly good conditions, we could not see any trace of the nucleus. It must have been below the 16th magnitude at that time.

In all the observations of this nebula the eyepiece was adjusted for the nebula itself, the focus of which, as I have shown elsewhere (*M.N.*, lx. pp. 255–257, and *Astrophysical Journal*, xiv. pp. 151–157), is about a quarter of an inch outside that for a star.

Following are the observations of the nucleus, taken from my notes on the nebula with the large telescope.

Observations of the brightness of the Nucleus.

1897 Sept. 17. Julian day 4185, $10^h\ 0^m$. The nucleus is 15th or 16th magnitude.

Sept. 28. 4196, $10^h\ 30^m$. The nucleus is seldom seen on account of poor seeing.

Oct. 3. 4201, $8^h\ 0^m$. The nucleus is seen only once in a while. The seeing is bad.

1898 Sept. 20. 4553, $9^h\ 30^m$. The nucleus is either absent or excessively faint. I think I can occasionally see faint traces of it. From recollection alone, it seems this must be very much fainter than it was last year. The definition is fair.

Sept. 26. 4559, $12^h\ 35^m$. I think I can see the feeblest trace of the nucleus, but it must be excessively faint. The seeing is fair. $13^h\ 0^m$. Can see it faintly with 700 diameters.

1899 July 31. 4867, $14^h\ 0^m$. The nucleus is very difficult.

Aug. 1. 4868, $13^h\ 30^m$. The nucleus is very difficult. The inside of the inner ring is very slightly darker than the general surface of the nebula.

Aug. 14. 4881, $15^h\ 10^m$. Can only catch the feeblest glimpses of the nucleus. It seems to me that it was brighter when the former measures were made. The inner ring is broken in two opposite places.

Aug. 26. 4893, $11^h\ 40^m$. The nucleus is very hard to see. The seeing is very bad.

Sept. 3. 4901, $9^h\ 30^m$. Can once in a while see the nucleus. The seeing is very bad.

Sept. 25. 4923, $15^h\ 20^m$. The nucleus is seen only feebly.

1900 July 17. 5218, $14^h\ 30^m$. The nucleus is 12th magnitude, and is very easy. It is yellowish.

July 28. 5229, $14^h\ 0^m$. I cannot see the nucleus; possibly could catch feeble glimpses of it. It ought to be visible. I have thought several times that the nucleus of this nebula is variable. It looks strongly that way now.

Aug. 6. 5238, $14^h\ 0^m$. The nucleus is fairly distinct. There is a $15{\cdot}5^m$ or 16^m star exactly on the north edge of the nebula.

Sept. 20. 5283, $10^h\ 10^m$. The nucleus is extremely faint. The seeing is fairly good.

1901 Aug. 7. 5604, $12^h\ 30^m$. The nucleus is readily seen. It is yellowish, and of the 12th magnitude.

Aug. 24. 5621, $16^h\ 0^m$. Can feebly see the nucleus, but it seems very faint. The sky is poor, but the nucleus ought to be brighter; the sky is too poor to be certain. I have suspected the nucleus of being variable. Cannot see it well enough to measure its position.

Dec. 22. 5741, $10^h\ 0^m$. Can faintly see the nucleus, but not distinctly enough to measure.

1904 Aug. 6. 6699, $13^h\ 0^m$. Professor Turner and I looked for the nucleus. We could see no trace of it with 460 or 700 diameters. The seeing was fair, so that we could see the details of the nebula well. The nucleus must be variable.

Aug. 8. 6701, $14^h\ 20^m$. Have just tried the focus, etc. for the nucleus. Can see nothing of it. Tried eyepieces 460 and 700. The seeing part of the time was fair.

Aug. 13. 6706, $11^h\ 40^m$. Can see the nucleus quite distinctly. It is about the 14th magnitude and is quite stellar. I think we ought to have seen it on the 6th if it was as bright as now. Seeing 2 to 3.

Aug. 15. 6708, $12^h\ 0^m$. Can faintly see the nucleus, though the sky is thick.

Aug. 16. 6709, $14^h\ 0^m$. Can very faintly see the nucleus; eyepiece 700. I am not sure but that it may be fainter than last night, for the sky, though poor and thick, is better than then.

Aug. 22. 6715, $10^h\ 0^m$. Can see the nucleus very plainly; eyepiece 700. It is 13th or 14th magnitude. Very much plainer than the central star in M 57. It is quite bright. Seeing 3–4.

Aug. 27. 6720, $13^h\ 30^m$. Eyepiece 700. The nucleus is well seen, about $12\frac{1}{2}$ magnitude; the same brightness as star *b* (more carefully:—It is decidedly brighter than *b*). It is about midway between *a* and *b* in brightness and is very easy. Perhaps slightly yellowish.

Aug. 29. 6722, $14^h\ 10^m$. By glimpses can see the nucleus. Can just see *b* on account of poor seeing, so the nucleus is not fainter than on the 27th. Seeing very bad.

Sept. 3. 6727, $9^h\ 0^m$. Can see the nucleus, but it is not clear. It is about 14th magnitude. Decidedly less noticeable than the star *b*. Eyepiece 700.

Sept. 5. 6729, $13^h\ 40^m$. Cannot see the nucleus, or if it is seen it is excessively faint. I think in the moments of good seeing I ought to see it. At such times *b* is quite easy or rather conspicuous. Just in this direction the sky seems fairly clear.

Sept. 10. 6734, $9^h\ 30^m$. Can feebly catch glimpses of the nucleus, but the sky is thick. I can see the star *b*, but not very distinctly, on account of thick sky. Eyepiece 700.

Sept. 12. 6736, $11^h\ 45^m$. The nucleus is very faint, 15th magnitude. Seeing 3. The star *b* is conspicuous.

Sept. 16. 6740, $10^h\ 0^m \pm$ (no record of time). The nucleus is easy, say $13\frac{1}{2}$ magnitude. It is less than the star *b*.

Sept. 24. 6748, $12^h\ 30^m$. Eyepiece 700. Can see the nucleus steadily. When best seen it is brighter than the star *b*. It is not less than 13th magnitude. It is perhaps as bright as 12 or $12\frac{1}{2}$ magnitude. Full moonlight. Looking more carefully, it is decidedly brighter than *b*. It is not fainter than 12th magnitude. Seeing 3.

Oct. 1. 6755, $9^h\ 30^m$. The nucleus is seen occasionally.

It is much fainter than the star *b*. I should say it is 14th magnitude. Eyepiece 700. Seeing 2.

Oct. 11. 6765, 9^h 30^m. The sky is thick. Cannot be sure of the brightness of the nucleus. It is faint, but the magnitude uncertain, say 14th or less. Seeing poor = 2.

Oct. 15. 6769, 13^h 40^m. Cannot see the nucleus. The star *b* is easy and conspicuous. I do not think the seeing is such as to blot the nucleus out if as bright as it has sometimes been. Eyepiece 700.

Oct. 17. 6771, 7^h 35^m. Can see the nucleus quite easily, 13th or 14th magnitude. I am sure it is brighter than on the 15th. It seems nearly as conspicuous as the star *b*. Seeing 2 or 3.

Oct. 22. 6776, 14^h 5^m. I think I can see the nucleus occasionally, but cannot be certain. If it is seen it is about as bright as the star *b*, which is faint and not well seen, but the seeing is too poor to decide. The nebula is 5 hours west.

Oct. 29. 6783, 7^h 30^m. I can see the nucleus. It is about 14th magnitude. By glimpses it is fairly well seen but I do not think it is as bright as it has been. Seeing 2. Eyepiece 700.

Oct. 31. 6785, 7^h 30^m. The nucleus is fairly well seen, about $13\frac{1}{2}$ magnitude. It is not as bright as the star *b*. A little later :—I see it better now. It is 13th magnitude. Eyepiece 700.

Nov. 5. 6790, 10^h 50^m. Eyepiece 700. I can see the nucleus occasionally. It is faint, perhaps 14th magnitude. The star *b* is quite easy and bright. I think the conditions are such that the nucleus ought to show up distinctly.

Nov. 12. 6797, 9^h 20^m. Eyepiece 700. I can see the nucleus occasionally fairly well. Seeing poor, but by moments of steadiness it must be as bright as 13th magnitude. It is nearly as conspicuous as the star *b*.

Nov. 14. 6799, 6^h 0^m. Eyepiece 700. It is faint, say 14th magnitude. I think it ought to be brighter if it is not variable. Though I can see it, it does not appear as stellar as it has appeared at times. There is a half moon shining, which may have something to do with it. Seeing = 3.

Nov. 26. 6811, 9^h 15^m. Eyepiece 700. The nucleus is faint. It is not brighter than 14th magnitude. By moments the seeing is fairly good. It is far less bright than star *b*. It is not well seen.

Dec. 5. 6820, 5^h 30m. Eyepiece 700. I can see the nucleus, but it is faint. It is between 13th and 14th magnitude, and is much less conspicuous than the star *b*.

Dec. 10. 6825, 7^h 0^m. The sky is too thick and the nebula is dull. I can see the star *b* faintly, but cannot see the nucleus. The sky is too thick, however, for this observation to count.

1905 Oct 28. 7147, 7^h 30^m. The nucleus is steadily seen with both eyepieces, perhaps best with eyepiece 460. It is nearly as bright as the star *b*. Eyepieces 460–700. Seeing 3.

Nov. 26. 7176, 8^h 35^m. The nucleus is very faint, very much fainter than the star *b*. Seeing 3.

Nov. 28. 7178, $8^h\ 45^m$. Eyepiece 700. The nucleus is very faint, but I can see it in spite of the bad seeing. It is much fainter than the star *b*.

1906 Aug. 11. 7434, $14^h\ 40^m$. Eyepiece 460. The nucleus is of about the same brightness as star *b*. Its appearance is the same with eyepiece 700. It is not less bright than the star *b*. Seeing poor.

Aug. 14. 7437, $11^h\ 50^m$. The nucleus is very faint. I think it is one magnitude less than the star *b*.

Sept. 4. 7458, $13^h\ 50^m$. Eyepiece 700. The nucleus is bright and conspicuous, of about the 12th magnitude, and seems to be yellowish. The seeing is not especially good; it comes and goes.

Sept. 8. 7462, $13^h\ 45^m$. The nucleus is easy. It is $\frac{1}{2}$ magnitude less than the star *b*. I think it is a little less bright than at the last observation. It is 13th magnitude.

Sept. 11. 7465, $13^h\ 50^m$. Eyepiece 700. I can see the nucleus occasionally very distinctly. It is 13th magnitude. I do not think it is any different from the few last observations. It is much less bright than the star *b*.

Sept. 22. 7476, $13^h\ 20^m$. Eyepiece 700. I can feebly see the nucleus. I think it is faint to-night. It is 2 magnitudes or more less than the star *b*. I think there is no doubt of its variability. The seeing is fair, 2 or 3.

Oct. 13. 7497, $10^h\ 55^m$. I can scarcely see the nucleus, and then not distinctly. It is several times fainter than the star *b*. Eyepiece 700.

Oct. 30. 7514, $7^h\ 30^m$. The seeing is too poor to tell anything about the nucleus.

Nov. 13. 7528, $9^h\ 5^m$. The nucleus is very faint. It is $1\frac{1}{2}$ magnitude less than the star *b*. If as bright as I have seen it sometimes, it ought to be quite bright to-night. Seeing 3, fairly good.

1907 July 2. 7759, $14^h\ 40^m$. Eyepiece 700. I can see the nucleus fairly well, but the sky white from the moon. Nucleus is 13th magnitude. Seeing 3.

Aug. 11. 7799, $12^h\ 0^m$. Eyepieces 460 and 700. I can feebly see the nucleus. It is very faint and pale, and cannot be seen steadily. Cannot see it any better with eyepiece 460. Seeing 3.

Aug. 13. 7801, $11^h\ 35^m$. Eyepiece 700. I can possibly see traces of the nucleus. It is very faint. The seeing is poor, but the nucleus ought to be seen easily if as bright as it has appeared sometimes. Of course I always test for the best focus on this nebula. With eyepiece 460, with the best glimpses, the inside of the ring is dark, and I can perhaps feebly see the nucleus. Seeing 2.

Aug. 20. 7808, $11^h\ 0^m$. Eyepiece 700. Can faintly see the nucleus. Perhaps it is somewhat brighter than at the last observation (?).

Sept. 1. 7820, $10^h\ 55^m$. Eyepiece 700. Can see the

nucleus quite easily. It may be 13th magnitude. I think it is brighter than usual. Seeing 2.

Sept. 3. 7822, $13^h\ 50^m$. Eyepiece 700. Can see the nucleus distinctly, but it is faint, say $13\frac{1}{2}$ or 14 magnitude. Seeing 2 or 3.

Sept. 8. 7827, $10^h\ 30^m$. Eyepiece 700. Can see the nucleus once in a while, but it is very faint. Seeing 2.

Sept. 10. 7829, $13^h\ 0^m$. Eyepiece 700. Can see the nucleus but it is faint; perhaps not so faint as previously. It is hard to see, as the seeing varies so. The sky is thickish.

Oct. 1. 7850, $10^h\ 40^m$. Eyepiece 700. Can see the nucleus very faintly. Seeing 2.

Oct. 4. 7853, $8^h\ 30^m$. Eyepiece 700. Can see the nucleus faintly. The seeing is too bad to make much out of it.

Oct. 13. 7862, $10^h\ 10^m$. It is bright, 11th or 12th magnitude, and yellowish. I do not think the good seeing is responsible for the brightness of the nucleus. It is perhaps slightly brighter than the star *b*. With eyepiece 460 it is bright and yellow. It is $\frac{2}{10}$ magnitude less than the star *a*. It is almost as bright as that star, perhaps just as bright. It is a conspicuous object, and could be seen with almost the worst seeing.

Nov. 3. 7883, $9^h\ 0^m$. The sky is too thick to see the nucleus. I can, however, see the 3 stars that are north, following; the fainter ones are just visible. A little later; can see it faintly, but this does not mean that it is really faint. Eyepiece 460.

Nov. 12. 7892, $10^h\ 5^m$. With both eyepieces, 460 and 700, can see the nucleus. It is not faint, about as bright as star *c* through thin clouds. It would be easy on a good sky.

Nov. 25. 7905, $6^h\ 40^m$. Eyepiece 460. Can see the nucleus, but it is faint. Eyepiece 700. I can see it with this also, but it is faint. Sky clear. It must be $1\frac{1}{2}$ magnitude less than when I observed it on the 12th. Seeing 2.

Nov. 26. 7906, $8^h\ 0^m$. It is faint. It is just as it was last night.

Nov. 28. 7908, $5^h\ 50^m$. The nucleus is quite easy, though not bright. I think it is certainly brighter than on Nov. 26. The seeing is not any better than on that date. It is poor. Occasionally the nucleus comes out quite distinctly, though much fainter than I have seen it at times previously. Later:—I can see it quite distinct and steady at times, and think it is decidedly brighter than on the 26th. Eyepiece 460. With eyepiece 700 I can see it fairly well, but not so good. Seeing too poor.

Dec. 3. 7913, $8^h\ 50^m$. Can see the nucleus, but it is very faint. Eyepiece 460. I think it is fainter than at the last observation. It is fainter than any one of the stars following. Seeing 2.

Dec. 5. 7915, $7^h\ 0^m$. The nucleus is faint, but once in a while it is fairly distinct. It is much less than any of the stars north, following. Eyepiece 460. With eyepiece 700 it is very faint and difficult.

Dec. 12. 7922, 8^h 0^m. The disc of the nebula is sharply outlined all round. It is a beautiful full ellipse. The outline is faint on account of moonlight, but distinct. The faintest part of the disc to the edge is at the south end of the major axis. Seeing poor. Perhaps there is some haze.

The inner ring looks irregular, and is more square than elliptical. The inside of this ring is dark, but not black. It is decidedly darker than the general surface of the nebula outside the ring. This inner ring is greenish-blue, and about $1\frac{1}{2}''$ in thickness. There is a suggestion of warmth to the general disc.

The nucleus is faint. (Too much moonlight?) Seeing = 2. It is rather hard to see, and is about the brightness of *c* or less. The south preceding $\frac{1}{3}$ of the inner ring is faint. It is hard to tell just how this axis lies, because of the break in the ring and from bad seeing. Eyepiece 700. Seeing = 2.

Dec. 25. 7935, 6^h 20^m. Nucleus fairly well seen. Seeing = 3. It is less bright than either of the three stars north, following. There is a 14^m star in the direction of *b*, whose estimated position with reference to the nucleus is P.A. $48° \pm$, dist. $87'' \pm$. There are several faint stars about $1'$ north, preceding the nebula.

The nucleus is not nearly so bright as I have seen it. The break in the following part of the inner ring is exactly opposite the break in the preceding side.

7^h 20^m the nucleus is quite bright and easy. Power 700. It is about as bright as *b* or *c*—perhaps less than these stars. It is slightly yellowish. The same results with eyepiece 460. Can feebly see the small star in the north, following edge of the nebula. I think the nucleus is less than either *b* or *c* by, say, $\frac{1}{2}$ magnitude. It is well seen occasionally. Carefully examined about *a*. There is no star nearer to it than *b*. The magnitudes of the three stars north, following, were independently estimated,—

$a = 12{\cdot}6$ magnitude.
$b = 13{\cdot}8$,,
$c = 13{\cdot}7$,,

The nebula was compared in the finder with several B.D. stars near it for brightness. The results were $8{\cdot}8^m$, $8{\cdot}6^m$, $7{\cdot}2^m$. Its ill-defined condition makes estimates of its magnitude rather uncertain. The mean of these, $8{\cdot}2^m$, will not be far out. This, of course, is on the B.D. scale of magnitudes.

1908 Jan. 2. 7943, 7^h 0^m. The nucleus fairly well seen. It is less than *b* by perhaps $\frac{1}{2}$ magnitude. Seeing = 3.

The notes sometimes seem to be conflicting as to the relative brightness of the nucleus—where an estimate of its magnitude would make it brighter than the comparison star, at the same time a statement is made that it is less than the star. This can only be explained by the difficulty of making any definite estimate of the magnitude of the nucleus, which is involved in a nebulous background, where the conditions of seeing, etc. affect its appearance far more than they do that of a star on the open sky.

Focal Measures of the Nebula and the Nucleus.

Following are the measures of the relative focus of the nebula and a star. These are readings from a scale of inches on the focussing tube.

1899 Aug. 1.	The nebula	2·54	(3)	inches
	A star	2.38	(3)	„
	Nebula—star +	0·16		inch
Sept. 25.	The nebula	2·39	(5)	inches
	A star	2·20	(4)	„
	Nebula—star +	0·19		inch
Oct. 7.	The nebula	2·49	(5)	inches
	A star	2·29	(5)	„
	Nebula—star +	0·20		inch
Oct. 15.	The nebula	2·60	(4)	inches
	A star	2·34	(6)	„
	Nebula—star +	0·26		inch
1907 Oct. 13.	The nebula	0·42	(6)	inch
	A star	0·20	(5)	„
	Nebula—star +	0·22		inch

For a comparison of the nucleus with the nebula and the star we have—

1899 Oct. 7.	Nebula	2·49	(5)	inches
	Nucleus	2·43	(5)	„
	Nebula—nucleus +	0·06		inch
	Nucleus	2·43	(5)	inches
	Star	2·29	(5)	„
	Nucleus—star +	0·14		inch
1907 Oct. 13.	Nebula	0·42	(6)	inch
	Nucleus	0·30	(5)	„
	Nebula—nucleus +	0·12		inch
	Nucleus	0·30	(5)	inch
	Star	0·20	(5)	„
	Nucleus—star +	0·10		inch

These give the following final values, weighted by the number of measures :—

Nebula—star
+0·21 inch

Nebula—nucleus
+0·09 inch

Nucleus—star
+0·12 inch

The plus sign indicates an increase of focus. From this we see that the focus of the nebula is 0·21 inch outside of that for the star, while it is only 0·09 inch outside of that for the nucleus. The focus for the nucleus is 0·12 inch outside of that for the star.

The readings on 1907 Oct. 13 were by a millimetre scale on the eyepiece. They have been converted into inches. The comparisons have been made with the star *a*.

The following measures were made of the dimensions of the nebula and of the positions of the axes of the larger and smaller ellipses.

For the Outer Ellipse.

	P.A. Maj. Axis.	Major Diameter.	Minor Diameter.
	°	″	″
1897 Sept. 17	34·9	30·64	26·08
20	36·4	31·55	25·94
21	32·9	30·08	25·41
1907 Dec. 12	35·4	29·75	25·83
	°	″	″
	34·9	30·50	25·81

For the Inner Ring.

	P.A. Maj. Axis.	Major Diameter.	Minor Diameter.
		″	″
1897 Sept. 21	...	13·58	11·16
28	43·0	...	...
1907 Dec. 12	54·4	15·10	11·32
	°	″	″
	48·7	14·34	11·24

These last results are discordant. It is difficult, from the nature of the inner ring, to measure its position angle.

Following are the measures of the three stars north, following the nebula.

The Nucleus and Star a.

	P.A.	Dist.	
1897 Sept. 17	62°·48	51″·31	from nucleus.
20	63·37	51·52	,, nucleus (?).
21	63·48	51·32	,, nucleus (?).
1900 July 17	63·07	51·24	,, nucleus. $a = 12^m \pm$
1901 Aug. 7	63·17	51·38	,, nucleus.
1904 Sept. 24	63·20	51·34	,, nucleus.
1907 Dec. 25	63·55	51·26	,, nucleus.
Mean	63°·19	51″·34	

The Nucleus and Star b.

	P.A.	Dist.	
1897 Sept. 28	45°·68	62″·76	from centre of nebula.
Oct. 3	46·33	63·01	,, centre of nebula (?). $b = 15^m \pm$.
1907 Dec. 25	46·92	62·12	,, nucleus.
Mean	46°·31	62″·63	

The Nucleus and Star c.

	P.A.	Dist.	
1897 Sept. 28	14°·81	68″·68	from centre of nebula.
Oct. 3	15·47	68·31	,, centre of nebula (?). $c = 14^m \pm$
1907 Dec. 25	14·94	67·38	,, nucleus.
Mean	15°·07	68″·12	

Photographs of the Nebula.

During the years 1899 and 1900 I made some experimental photographs of a few of the brighter planetary nebulæ, using a Cramer Crown plate, which was very sensitive, and a yellow colour-screen, the same one that had been used in the regular star photographs. The results, though not entirely satisfactory, were very gratifying. The great difficulty lay in the very long exposures that were required. The planetary nebula in Draco, N.G.C. 6543, could be secured in from twenty minutes to an hour, while the others tried by me required several hours, and were then under-exposed. In the case of several of these nebulæ remarkable details were shown that were not visible to the eye in the great

telescope. Especially astonishing were the photographs of the Draco Nebula, which showed it to have a delicate spiral structure.

Among the nebulæ thus photographed was N.G.C. 7662. In none of the photographs of this object was the exposure time long enough to bring out all the fainter parts of the nebula. The inner ring came out very strong, showing the breaks just as I had seen them in the telescope and drawn them. There were some thread-like lines (which I have not seen visually) running from the inside of the ring to the nucleus. On these photographs the nucleus does not seem to be central in this inner ring, but is perceptibly displaced to the following side. The main disc of the nebula is decidedly uneven in brightness. The north preceding portion is very faint, as is also the north following side, but less so. A section of the south following edge is very bright, while the opposite side of the nebula is somewhat more luminous than the general surface. A section of the north preceding outline of the nebula is lacking for want of longer exposure. There is evidence in the photographs of changes of brightness in the nucleus.

In all these photographs the plate was placed from $\frac{1}{4}$ to $\frac{1}{3}$ of an inch outside the stellar focus, in conformity with my previous observations, which showed that the focus for the nebulæ is outside that for a star. See *M.N.*, lx., January 1900, and *Astrophysical Journal*, xiv., for October 1901.

I hope soon to be able to take up this subject again, with the large telescope, with suitable screen and plates prepared by Mr. Wallace.

Following is a list of the photographs.

1900 Aug. 18. Exposure $0^h\ 35^m$. Exposure stopped by storm. Shows feeble traces of nucleus, inner ring and part of disc.

Aug. 21. Exposure $1^h\ 18^m$. Sky poor. Seeing poor. Faint traces of nucleus. Shows inner ring and bright spot on s.f. edge.

Aug. 28. Exposure $0^h\ 45^m$. Bad sky ending in fog. Shows nucleus feebly. Only inner ring feebly shown.

Sept. 3. Exposure $1^h\ 5^m$. Sky poor.

Sept. 3 (same date). Exposure $1^h\ 30^m$. Seeing fine, sky good. The last one is best. They are both strong and show nearly all of the nebula. The nucleus is strong in both.

Sept. 20. Exposure $2^h\ 35^m$. Sky very clear. Seeing poor. Shows inner ring and part of entire disc.

Sir John Herschel's Observations.

In a paper containing observations of the nebulæ made with the 20-feet reflector at Slough from 1825 to 1833, read by Sir John Herschel before the Royal Society, November 21, 1833, the following observations of this nebula are recorded :—

"Sweep 183. A fine planetary nebula. Diameter 12″; with 240 beautifully defined, light, rather mottled, and the edges the least in the world unshaped. It is not nebulous, but looks as if

it had a double outline, or like a star a little out of focus. A perfect circle. Has a star near; pos. 68°·1, well measured over the centre of the nebula. See fig. 45.

"Sweep 180. *v* B; R; bluish white; 8 or 10″ diam. It has no haze at the edges, but ? if it be not enveloped in an *e* F nebulosity (perhaps arising from glare), and also the star 13 m., whose pos. is 61°·9 and Δ R A = 4ˢ·0. The light is a very little mottled, and not absolutely planetary. It is = a * 7 m. (*N.B.*—These *satellites* of planetary nebulæ ought to be especially attended to.)

"Sweep 204. Diameter in R A = 2ˢ·0 of time. The attendant star is double A = 12 m; B = 13 m., pos. of A = 58°·5; Δ R A from centre = 4ˢ·0. The nebula is not perfectly round, light equal, but a very little hazy at the edges.

"Sweep 189. Exactly R; 2ˢ·5 of time = diam. in R A. Has a * 13 m. pos. 69°·0; dist. 30″; the light of the disc is perfectly equable, and equals a star 8 m.

"Sweep 190. Exactly R; = a * 8 m; a * 13 m n f; pos. = 65°·6 by microm.; dist. = 30″. [*N.B.*—The great discordance in the measured positions of the satellite-star seems to arise from the star being double, and taken for a single one. Whenever this occurs, it is a source of error, and should be most carefully guarded against in all future observations of this interesting object.]"

I have carefully examined, with the 40-inch telescope, the immediate vicinity of the star *a*—the one referred to by Herschel as being double. There are no signs of duplicity, and there is no star nearer than *b*. This statement rests specially on observations made 1907 Dec. 25—a good night. From the recent measures there seems to be no certain motion in any of the stars *a*, *b*, and *c*, or the nucleus.

Observations with the Rosse Telescope.

In *Phil. Trans.* for 1850, p. 513, the Rosse observers, using the great reflector, give the following account of their observations of this nebula:—

"Oct. 31, 1848. Has a central spot, at moments very dark.

"Dec. 13, 1848. Nothing more, except perhaps that faint external annulus extends further than had been seen before.

"Dec. 14, 1848. Note by Mr Johnstone Stoney:—'Three stars near it, somewhat in this fashion; showed it to Sir James South.'

"Dec. 16, 1848. Sketches made by Lord Rosse and Mr Johnstone Stoney.

"Dec. 19, 1848. Drawing confirmed."

A drawing of the nebula is given, plate xxxviii. fig. 13. This does not show any central star.

In *Phil. Trans.* for 1861, plate xxx., another drawing appears, fig. 40, which differs very greatly from the first one. This shows

the central star conspicuously. The following additional notes are also given:—

"Since the publication in the *Transactions* for 1850. The outlying portions in the published sketch are parts of spiral branches. Fig. 40, plate xxx. represents it as seen on a very fine night (Sept. 16, 1852), with a freshly polished speculum which defined very sharply. Oct. 2, 1856. All the details in Mr Stoney's drawing very well seen. Oct. 16, 1857 [J.D. 2, 399, 604]. The spiral arms and the * in centre distinctly seen."

The drawing referred to shows a nearly round disc with a brighter spiral arrangement commencing near the edge and ending near the central star. This light spiral is outlined by a dark shading, or by a parallel dark spiral. I do not find any reference to previous observations of the nucleus by the Rosse observers, though, from the last sentence quoted, they seemed to be familiar with the star.

Other Observations.

Professor Burnham (*Pub. L.O.*, ii.) measured the central star on two nights with the 36-inch telescope:

1890·78 62°·9 51″·84

The central star was 15th magnitude and *a* 13^m. He also gives measures by O. Struve:

1847·86 61°·3 51″·85

Struve could not see the central star. His measures were, therefore, referred to the centre of the nebula.

Volume 13, Annals of the Harvard College Observatory, pp. 80–81, contains some observations of this nebula by Winlock and Searle about 1866. Combining their various measures of the dimensions, etc. of the nebula we get—

P.A. major axis of the nebula 47° ±
Diameter of the major axis 27″·4 ±
Diameter of the minor axis 24″·4 ±

Some of the notes in these observations are:—"Edges as definite as Jupiter's. Two other observers also considered the colour to be green, much deeper under illumination than without . . . . Two edges seen . . . . as of a brighter nebula placed over a dimmer one. The brighter and fainter ellipses have major axes slightly differing in direction; the brighter having the position angle of its major axis 10° to 15° the larger (J. W. obs.) 20° to 30° according to a less careful estimate by G. M. S., to whom the northern side appears brighter; appearance to J. W. that of a hollow cylinder with one end turned nearly towards the observer."

Searle and Winlock also measured the position angle and distance of the star *a* on two nights in the last part of 1866,

P.A. 62°·7 Dist. 52″·14

The central star was not seen at any time.

The Probable Period of Variation.

An investigation of the observations seems to give a period for the variability of the nucleus, or central star, of about 28 days. The observation of 1906 Sept. 4, however, does not seem to fit in with this period.

I give below some efforts to determine the period, utilising the early Rosse and Lassell observations in connection with my own. In general the results imply a period of 28 days. It is my intention to follow the variations of the nucleus more closely, and to determine accurately the period and the light curve. I think this is the first case in which the nucleus of a planetary or other nebula has been shown to certainly vary in its light. It is also my intention to follow up some of the other planetary nebulæ, for there are one or more cases where I have suspected variability in the nucleus.

If we take the observation of 1907 Oct. 13 and combine it as below we shall get the following results:—

Taking the observations of 1900 July 17, with an interval of 94 periods, we shall have the period 28·1 days.

If we use the observation of 1901 Aug. 7, with an interval of 80 periods, we shall have for the period 27·9 days.

If we combine the observation of 1904 Aug. 27, with an interval of 41 periods, the result is period, 27·9 days.

If we use the observation of 1904 Sept. 24, with an interval of 40 periods, we have period, 27·9 days.

The observation of 1906 Sept. 4, with an interval of 14 periods, gives a period of 28·9 days.

The observation of 1906 Aug. 11, gives, with an interval of 15 periods, 28·7 days.

In *Memoirs* of the R.A.S., vol. xxxvi. p. 51, Lassell, gives observations of this nebula with his 4-foot reflector at Malta on 1862 Oct. 23 (Julian day 2,401,437).

"Central point not stellar, but apparently a very minute planetary disk of a bluish colour. . . . . The central point is bright and conspicuous, not to be overlooked even in a cursory examination."

Assuming that the nucleus was at its maximum brightness at or near this observation, by comparison with the observation of 1904 Sept. 24 (J.D. 2,416,748) when it was at or near maximum, we have an interval of 547 periods, which gives a period of 28·0 days.

If we compare this observation with that of 1906 Sept. 4 (J.D.

2,417,458), 579 periods would have elapsed, which would give a period of 27·7 days.

Or taking the observation of 1907 Oct. 13 (J.D. 2,417,862), when it was evidently close to maximum, there will have elapsed 587 periods from which the period derived is 28·0 days.

For some reason all the comparisons with 1906 Sept. 4 give discordant results.

The Rosse observation of 1857 Oct. 16 (J.D. 2,399,604) was undoubtedly at or near maximum. Combining this with my observation of 1907 Oct. 13, assuming an interval of 652 periods, we shall get a period of 28·0 days.

The accompanying drawing of this planetary nebula (Plate 12) was made with the 40-inch refractor. I think it is a fair representation of the object.

In a paper read before the Royal Dublin Society (*Transactions*, vol. ii. (Series ii.)) on Feb. 18, 1878, the following additional notes from Lord Rosse are given:—

"1848 Oct. 31. Has a central spot, at moments very dark.

Dec. 13. Nothing more, except perhaps that F external annulus extends farther than had been seen before.

Dec. 15. 3 st. near it. Showed it to Sir J. South.

Dec. 16. Sketches made by Lord R. and myself.

Dec. 19. Drawing confirmed. [P.T., 1850, plate xxxvii. fig. 13.] This should be plate xxxviii.—(E. E. B.)

1852 Sept. 16. Very fine night, freshly polished speculum which defines very sharply. Drawing made [P.T. 1861, plate xxx. fig. 40]. In the figure in P.T., 1850, the outlying portions are parts of spiral branches.

1856 Oct. 2. All the details of Mr. B. Stoney's drawing very well seen by Lord R. and myself.

1857 Oct. 16. The spiral arms and the star in centre distinctly seen.

1872 Oct. 7. Moderately well defined, an outer F atmosphere; darker in the middle about the spiral.

1873 Aug. 20. Has a *13 m in Pos. 244°·8. Dist. 46″·6.

Sept. 28.	$a\beta$	Pos. 356°·1	Dist. 25″·7.
	$\beta\gamma$	,, 309·1	,, 44·3.
	N a	,, 61·2 (4).	

From a to limbs, mean 64·4 $\beta = \gamma = 16$ m.
Distances measured by Lord R. by C [Dr Copeland].

a to far limb	67″·2	64″·2
a to near limb	40·5	34·1
a to centre	51·3	48·9

Oct. 11. *8 m. Pos. 80°·0 (3). Dist. 498″·9 (2). Faintest of 3 st. near [γ]. Pos. 14°·2. Dist. from limbs 82″·1 and 51″·2. Certainly no conspicuous nucl. (xxxii. obs.)."

The position angle 244°·8 is 180° too great. It should be 64°·8, for the measures can only refer to the star *a*. The distance is too small.

A sketch accompanies the notes with the great reflector, p. 174. This shows the nebula and three stars a, β, γ, following it. I have identified these stars with mine, as below :—

$$a = a$$
$$\beta = b$$
$$\gamma = c$$

I have computed, for comparison, the position angles and distances $a\beta$, $\beta\gamma$ and $a\gamma$ from my measures. They are :—

			Residuals R − B	
	°	″	°	″
$a\beta$ (ab)	358·7	20·20	− 2·6	+ 5·5
$\beta\gamma$ (bc)	309·2	35·59	− 0·1	+ 8·7
$a\gamma$ (ac)	326·7	51·30		

I do not think these large residuals indicate any real change in the relative positions of the stars. In this case, the position angles with the large reflector seem to be fair, but the distances must be affected with very serious errors.

It will be seen that the presence and absence of the central star, in the Rosse observations, are strongly attested by the notes of 1857 Oct. 16, and 1873 Oct. 11.

Yerkes Observatory,
Williams Bay, Wisconsin :
1908 *February* 6.

P.S., 1908 *April* 21.—In reference to the variability of the nucleus of the nebula N.G.C. 7662, I have carefully gone over all the literature of the subject, and in no case has there been any suggestion of variability (outside of my own suggestions). There does not seem to have been any suspicion, even, of variability.

S

N

The Planetary Nebula N.G.C. 7662.—Drawn by E. E. Barnard.

[The half-tone is too harsh. This causes the central darkness to be a little too great, relative to the general disc. It also makes the bright, inner ring too bright. The south portion of this ring should be shaded or slightly darkened, as the north portion is somewhat the brightest part of the ring.—E. E. B.]

Note on the Period of Variation of Barnard's Variable Nebula in Andromeda. By Professor H. H. Turner.

At Professor Barnard's request, I add the following note on the period :—

Selecting the well-determined maxima shown in the first column of the following table, the intervals can be represented by multiples of $27\frac{1}{3}$ days, as shown in the next three columns. In the last column is shown the approximate alteration of period when the multiple is altered by one whole unit.

Epoch of Maximum.	Interval.		Period.		Multiple.	Diff. for 1 Unit.
2399604	d		d			d
(Rosse)	1833	=	27·36	×	67	0·4
2401437						
(Lassell)	13781	=	27·34	×	504	0·05
2415218	386	=	27·57	×	14	2·0
5604	1116	=	27·22	×	41	0·65
6720	28	=	28	×	1	...
6748	686	=	27·44	×	25	1·0
7434	24	=	24	×	1	...
7458	404	=	27·0	×	15	2·0
7862						
	2644	=	27·26	×	97	

It is clear that the long interval of 13781 days is of little use for defining the period at present; but Barnard's observations and the interval between Rosse and Lassell give about $27\frac{1}{3}$ days.

But the last column shows that, since all the differences are approximate sub-multiples of 2 days, periods of 29·3 or 25·3 days might suit the observations. Below are given the corresponding figures :—

Interval.		Multiple.		Period.		Multiple.		Period.
1833	=	62	×	29·57	=	72	×	25·46
386	=	13	×	29·69	=	15	×	25·73
1116	=	38	×	29·37	=	44	×	25·36
686	=	23	×	29·83	=	27	×	25·41
404	=	14	×	28·9	=	16	×	25·25
4425	=	150	×	29·50	=	174	×	25·44

The accordance for the 29 day period is, however, not good. The periods $27\frac{1}{3}$ and $25\frac{1}{2}$ days divide approximately into 355 days,

so that at the same time of year they run together nearly for several succeeding years.

Hence the observations do not readily discriminate between them; but those at 2416820, 2417176, and 2417178, when the nucleus was faint, seem to favour the $27\frac{1}{3}$ day period, since they make the epochs 10 or 12 days away from maximum, while the $25\frac{1}{2}$ day period makes them close to maximum. Professor Barnard proposes to make further observations to settle the question.

Corrections to Professor Turner's Paper "On the Classification of Long-period Variable Stars, and a possible Physical Interpretation." By the Rev. J. Stein, S.J., Sc.D.

(*Communicated by Professor H. H. Turner, D.Sc., F.R.S.*)

[My best thanks are due to Father Stein for pointing out the corrections noted below. According to the general permission given in the last sentence of his letter, I publish it at once.—H. H. T.]

Specola Vaticana, April 3, 1908.

Dear Professor Turner,—Lately I have studied your remarkable article on the "Long-period Variable Stars" (*M.N.*, lxvii. p. 332). My interest was the greater as this is the first serious attempt to bring the long-period light-curves in close relationship to the laws of sun-spot variation.

I take the opportunity for suggesting some ideas by which the number of possible cases in your hypothesis may be still more reduced.

1°. Is there not a little oversight in your paper on page 345? There it is said—

"The mean (foreshortening) factor for the spot may be taken as

$$\int \cos\zeta \, dZ = \ldots .$$"

Should this not be

$$\frac{\int \cos\zeta \, dZ}{\int dZ} \; ?$$

Then "the total effectiveness" would be (*top line of page* 346)—

$$s = \frac{1}{2\pi}(4\beta \cos\epsilon \sin\lambda + 4 \cos\beta \sin\epsilon \cos\lambda),$$

and in the "polar view"—

$s_1 = \sin\lambda$ (as correctly stated on page 342, line 6; but not as incorrectly on page 346).

In the "equatorial view"—

$$s_2 = \frac{2}{\pi}\cos\lambda \quad (\textit{instead of } 4\cos\lambda \textit{ on p. } 346).$$

Thus $s_1 = s_2$ for $\tan\lambda = \frac{2}{\pi}$, or

$$\lambda_0 = 32^\circ{\cdot}5,$$

and λ_0 *is defined without any ambiguity*, such as is contemplated on p. 348. Of course, if we take a broad belt of sun-spots instead of an infinitesimal one, λ_0 will be slightly different from the given value. In the extreme case, when the whole sphere is all over covered with spots, the mean foreshortening factor will be $\frac{(\text{Disc})}{(\text{Hemisphere})} = \frac{1}{2}$, and $\lambda(=\lambda_0) = 30^\circ$. The variation of λ_0 being so slight, we can practically put $\lambda_0 = 30^\circ$ for all cases.

2°. The brightness at maximum being from 40 to 100 times that at minimum, it seems very probable that at maximum the whole surface of the star is in activity and covered with faculæ. This may be confirmed by what we know of the "polar type" of the Sun's corona at maximum, when the streamers are numerous in nearly all solar latitudes (*M.N.*, lxiii. p. 482), in opposition to the "equatorial" or "wind vane type" at minimum. In this hypothesis the maxima of polar and "equatorial view" must coincide, and this would seem a new argument for the more preferable "hypothesis of superposing maxima."

If these remarks are right, it follows that the latitudes given on page 348 for $\lambda_0 = 30^\circ$ are the most probable.

Now, starting from the equation $B = sF$, we can calculate F.

Subtracting the numbers of Table III. Group I. (the "polar view group") from unity, and multiplying by 1·6, we get

	Max.	2	3	4	5	6	7	8	9	10	11	12
$\log s_1$ F	= 1·60	1·39	·99	·63	·21	·00	·10	·48	·96	1·12	1·33	1·44
$\log \sin\lambda$	= 9·70	9·62	9·45	9·28	9·13	9·19	9·52	9·91	9·99	·00	9·99	9·93
log F	= 1·90	1·77	1·54	1·35	1·08	0·81	0·58	0·57	0·97	1·12	1·34	1·51
F	= 79	59	35	22	12	7	4	4	9	13	22	32
λ	= 30°	25°	16°	11°	8°	9°	19°	54°	79°	82°	78°	58°

The hypothetical law suggested by these numerical results can hardly be better expressed than in the words used by the Astronomer Royal for characterising the sun-spot phenomena from 1874 to 1902 (*M.N.*, lxiii. pp. 452–3). We have only to omit the words between brackets :— . . . "The years of maximum showed spots in practically every latitude [between 30° N and 30° S]. In the years following the maximum a marked tendency was shown for the spots to appear in lower latitudes . . . . about one or two years before minimum no spots were seen outside the limit of 18° from the equator. But immediately minimum was reached the

spots became more widely extended in latitude, owing to the occurrence of outbreaks in high latitudes. Thus at minimum each hemisphere, considered separately, showed two clearly defined spot-zones marked off from each other by a broad belt in which there were no spots at all. Of these two spot-zones in each hemisphere, the lower appears to correspond to the series of spots of the expiring cycle. During the periods of increase the equatorial belt was almost wholly free from spots, indicating possibly the complete disappearance of the last members of the old cycle. At maximum, however, the spots of the new cycle were most widely spread, and were even seen in the near neighbourhood of the equator . . . . ."

Finally, I wish to tender you my sincere congratulations on your original interpretation of the long-period variation, and to express the hope that my little remarks may contribute to enhance its evident plausibility.

It goes without saying, that you may dispose of these remarks at your own pleasure.—I am, very faithfully yours,

J. Stein, S.J.

Measures of Southern Double Stars in 1907. By James L. Scott.

The following measures of southern double stars were made with the same 5-in. refractor as those in *M.N.*, vol. lxiv. p. 52, bright wire illumination being used throughout. The weather during 1907 was, on the whole, distinctly poor, cloudy nights and bad definition being the rule rather than the exception; the number of stars measured was therefore smaller than usual.

Star's Name.	R.A. h m	S. Dec. ′	P.A. °	Distance. ″	Mags.		Nights.	Date. 1907.
Σ 3063	0 2	5 6	218·0	1·85	8½	10	3	·942
O. Stone 51	0 2	14 54	288·6	9·88	9	9	2	·942
β 391 (κ_1 Sculptoris)	0 3	28 32	270·8	1·15	6½	6½	3	·928
h 3375	0 29	35 32	166·3	6·05	6½	9	2	·868
O. Stone 3	0 47	23 9	267·2	2·10	8¾	9½	3	·953
h 3407	0 48	25 31	127·3	13·20	7	7½	2	·871
Ll. 1662	0 53	16 13	214·8	6·50	8	8½	2	·006
h 2036	1 15	16 20	13·3	1·48	7	7½	3	·956
τ Sculptoris	1 32	30 25	97·2	1·90	6	7	3	·953
Σ 231 (66 Ceti)	2 08	2 52	232·0	16·05	5¾	8¾	2	·956
Hastings I	2 11	18 42	356·1	2·01	8	8½	3	·956
θ Eridani	2 54	40 43	86·1	8·46	3¼	4¾	2	·953
h 3750	5 16	21 20	279·8	3·50	4¾	9½	2	·164
h 3752	5 18	24 52	102·7	3·25	5½	7½	2	·164

Star's Name.	R.A.	S. Dec.	P.A.	Distance.	Mags.		Nights.	Date.
	h m	° ′	°	″				1907.
Argelander (I)	6 02	25 1	295·0	4·74	8¼	8½	2	·205
h 3858	6 22	34 59	316·6	3·75	7½	8½	2	·205
h 3938	7 9	22 42	249·2	19·50	7½	8½	2	·205
Σ 1104	7 24	14 47	333·6	2·30	7	9	2	·246
h 3973	7 26	20 40	38·4	8·90	8½	9	2	·249
O. Stone 18	8 5	26 50	261·2	3·32	8½	9	2	·249
Dunlop 63	8 6	42 21	80·9	5·50	7	8	2	·246
γ Argûs (A & B)	8 6	47 3	219·8	41·52	2	6	2	·249
(A & C)	...	...	148·7	62·11	2	8	2	·249
h 4093	8 23	38 43	122·7	8·03	7	7½	2	·260
Howe 23	9 3	31 11	307·2	3·30	8¼	9	2	·260
h 4193	9 12	22 42	118·8	3·05	8	10	2	·249
H. C. Wilson	9 16	23 3	36·9	1·30	7¾	9¾	3	·249
ζ_1 Antliæ	9 26	31 27	210·2	8·20	5¾	6½	2	·260
h 4224	9 32	30 47	117·3	7·00	8	9	2	·249
C. G. C. 13,722	9 58	17 30	274·0	20·02	6½	7¼	1	·260
Howe 15	10 58	26 58	338·2	2·10	7·5	9·3	2	·271
17 Crateris	11 27	28 43	209·3	8·88	5½	5½	2	·378
h 4455	11 32	33 1	244·0	3·80	6¼	9¼	2	·378
C. G. C. 15,942	11 35	37 30	95·8	16·50	7	9	1	·397
h 4481	11 52	21 59	197·1	3·30	8	8	2	·378
h 4495	12 0	32 23	316·8	6·60	6½	9	2	·378
C. G. C. 16,612	12 4	34 6	202·5	3·75	6½	9	2	·378
Jacob 8	12 5	34 9	20·3	3·10	6½	9	2	·394
S 634	12 6	16 14	285·5	6·67	7	9	2	·378
D Centauri	12 9	45 10	242·4	3·08	5½	7	2	·394
h 4518	12 19	40 50	207·6	10·10	6¾	9	2	·446
δ Corvi	12 25	15 58	214·3	24·45	3	8½	1	·430
Σ 1669	12 36	12 28	306·0	5·70	6½	6½	2	·446
γ Virginis	12 37	0 54	326·8	6·01	3	3	3	·430
h 4556	12 49	27 25	80·0	6·50	7	8½	2	·397
O. Stone 27	12 52	12 36	66·9	1·95	8½	8¾	2	·446
h 4563	12 55	33 5	236·0	6·82	7½	8¼	1	·430
h 4587	13 20	42 32	86·3	5·20	8	8½	2	·446
Ö. A. 12,867	13 21	22 43	356·2	1·61	8·8	9	2	·446
β 114	13 29	8 6	143·0	1·50	8	8	2	·446
h 4608	13 36	33 28	177·5	4·60	8	8¼	1	·430
h 4617	13 45	29 23	260·8	5·05	7½	9½	2	·446
β 343	13 46	31 7	118·0	1·03	6	7	3	·446
κ Centauri	13 46	32 30	109·5	8·17	4¾	6	1	·430

Star's Name.	R.A.	S. Dec.	P.A.	Distance.	Mags.		Nights.	Date.
	h m	° ′	°	″				1907.
Howe 26	13 49	31 37	115·9	6·40	7½	10	2	·485
h 4661	14 6	38 35	230·8	4·48	9	9	2	·482
Σ 1837	14 19	11 13	300·4	1·40	7	8·5	2	·482
Howe 29	14 31	37 6	213·4	4·25	8	8½	2	·485
h 4690	14 31	45 42	23·6	19·50	5½	8	1	·501
54 Hydræ	14 40	25 2	130·2	8·71	5¼	7¼	2	·482
β 106 (A & B)	14 44	13 44	341·4	1·60	5½	6½	2	·485
P XIV., 212	14 52	20 57	296·8	17·35	6	7½	2	·482
h 4722	14 53	30 18	340·4	8·65	7½	9	2	·501
β 239	14 53	27 16	316·8	0·90	6	6½	3	·485
h 4727	14 57	27 26	216·7	7·38	8¾	8¾	2	·482
β 119	15 0	6 37	300·4	1·60	8	8½	4	·501
h 4743	15 1	32 32	196·9	11·19	7¼	7¼	2	·485
Howe 31	15 7	36 52	46·8	6·50	7¼	7¼	2	·485
h 4776	15 23	41 34	228·6	5·52	7	8½	2	·501
h 4783	15 25	19 48	280·1	11·05	6¾	8½	2	·498
Howe 37	15 36	14 28	90·7	5·43	8	8	2	·490
η Lupi	15 53	38 7	20·4	15·30	4	8	1	·490
Lal 29,720	16 13	19 49	21·2	13·18	8	8¼	1	·501
σ Scorpi	16 15	25 21	272·7	20·60	3	7½	1	·583
h 4845	16 17	41 01	135·6	2·10	8	8½	2	·548
h 4848	16 17	32 58	153·2	5·90	7	7½	2	·501
h 4850	16 18	29 28	350·8	6·30	6	6½	2	·501
ρ Ophiuchi	16 20	23 13	355·2	3·40	5¼	6¼	2	·501
Skinner	16 39	17 8	86·0	3·50	8½	8½	2	·605
P XVI., 236	16 51	19 23	231·3	4·80	7¼	8½	1	·583
Σ 2119	17 01	13 48	192·2	2·20	8	8	2	·548
36 Ophiuchi	17 9	26 27	187·8	4·18	5¼	5½	2	·548
β 958	17 10	19 14	213·8	1·55	8½	9	3	·548
38 Ophiuchi	17 11	26 31	337·6	5·68	6½	9	2	·548
β 416	17 12	34 53	273·0	2·30	6	8	4	·559
β 126	17 14	17 39	262·5	2·00	6¼	8¼	2	·559
Σ 2204	17 40	13 16	24·3	14·10	7	7·2	1	·583
h 5003	17 53	30 15	104·8	5·25	6	7	2	·605
τ Ophiuchi	17 57	8 11	258·2	1·94	5½	6	3	·559
Howe 50	17 59	36 35	2·6	3·05	7¼	8¼	2	·583
70 Ophiuchi	18 0	2 33N	164·7	2·61	4½	6	4	·559
β 245	18 4	30 45	353·5	4·15	6	8	2	·643
H N 125	18 23	25 6	101·6	3·01	7½	7¾	2	·605
S 715	19 12	16 09	14·6	8·30	7¼	7½	2	·698

Star's Name.	R.A.	S. Dec.	P.A.	Distance.	Mags.		Nights.	Date.
	h m	° ′	°	″				1907.
h 1381	19 12	16 10	195·7	4·95	8·1	8·3	2	·698
β 142	19 22	12 21	342·6	1·60	8	8	3	·698
Ho. 462	20 59	11 29	218·5	2·55	9	9½	3	·832
h 5252	21 7	15 25	318·2	3·02	8	8½	2	·832
C. G. C. 29,658	21 35	18 53	66·0	5·00	8	9·4	2	·846
η Piscis Australis	21 55	28 56	116·3	1·74	6	6½	5	·868
See 470	22 5	24 5	36·8	1·66	8	9	4	·868
S 808	22 19	20 52	151·8	7·00	7	8·3	2	·797
53 Aquarii	22 21	17 15	310·8	6·80	6½	6¾	2	·797
ζ Aquarii	22 24	0 32	315·2	3·05	4·5	4·6	4	·832
H N 117	22 34	28 52	64·3	3·04	7¾	8½	2	·871
γ Piscis Australis	22 47	33 24	266·4	3·56	4½	8¾	2	·871
Σ 3008	23 19	9 1	235·8	3·60	7½	8½	2	·871
ι_2 Aquarii	23 41	19 14	137·8	5·90	5½	7	2	·928
B.A.C. 8308	23 49	27 36	269·3	6·90	6¾	7½	2	·950
Σ 3046	23 51	10 03	252·7	3·18	8¾	9¼	3	·950

Shanghai:
1907 *December* 31

Note on the adopted co-ordinates of the Bombay (Colaba) Observatory. By A. M. W. Downing, D.Sc., F.R.S.

It may be desirable to point out that the relatively large change in the position of the Colaba Observatory, as given in the *Nautical Almanac* for 1909 and following years, from that given in the *Nautical Almanac* for 1896–1908 inclusive, arises from the large difference existing in that part of India between the geodetic and astronomical co-ordinates. I am indebted to the courtesy of the Headquarters Staff of the Trigonometrical Branch of the Survey of India for the following particulars as to the position of the Colaba Observatory:—

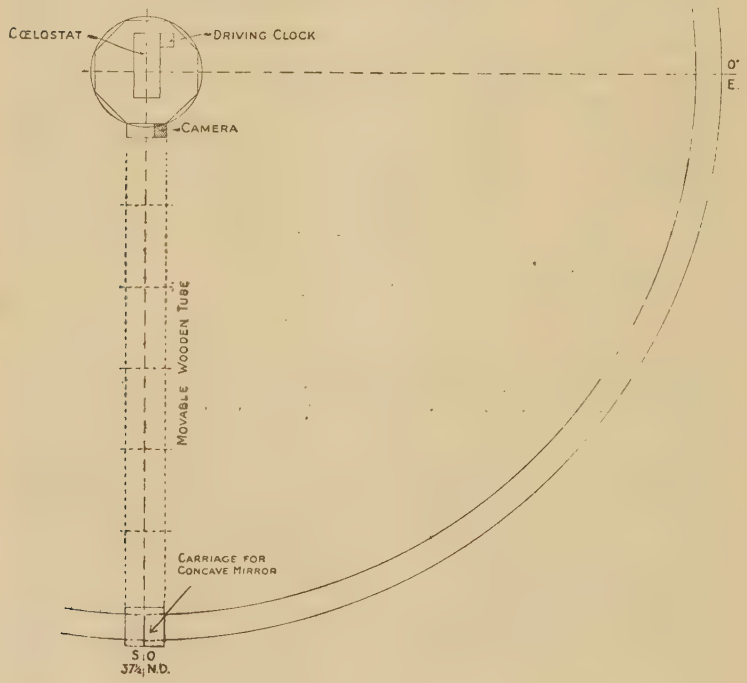
	Astronomical.	Geodetic.
Latitude,	18° 53′ 36″·18 N.	18° 53′ 46″·51 N.
Longitude,	$4^h\ 51^m\ 15^s{\cdot}72$ E.	$4^h\ 51^m\ 15^s{\cdot}15$ E.

It will thus be seen that the quantities given in the *Nautical Almanac* for 1909 and onwards are the astronomical latitude and the geodetic longitude. On the other hand, up to and including the current year, the quantities given are the geodetic latitude and the astronomical longitude; the values of these co-ordinates given above being later, and presumably more accurate, than those communicated to me in 1892, and which appear in the *Nautical Almanac* for the years specified.

Description of a 24-inch long-focus Cœlostat Reflector.

By J. H. Reynolds.

The idea of mounting a long-focus reflector in conjunction with a cœlostat was mentioned by Dr. Common in 1900, and it has been successfully carried out at Mount Wilson with the Snow telescope for spectroheliographic work. The present instrument is intended for use with a spectroheliograph which is in course of construction, but it is also arranged for photographing the Moon and planets, and



the brighter star clusters and nebulæ. For several reasons it was found advisable to keep the focal length within moderate limits; the concave mirror has therefore been made with a focal length of 38 ft., an enlarging lens being used when a greater scale is required.

The cœlostat is mounted in an octagonal wooden house, which revolves so as to face the various azimuths required; the roof revolves separately, and has a large opening covered with hinged shutters. The plane mirror, which is 28 in. in diameter, is furnished with a slow motion at right angles to the polar axis; this is very useful for getting the object in the centre of the field, and is practically indispensable for lunar work.

At the required distance from the cœlostat house a circular rail has been fixed for carrying the concave mirror. This rail runs from east to south, and gives a range in declination of $37\frac{1}{2}^{\circ}$. This, of course, is a limited range, and is the main defect of this form of mounting, especially for stellar and nebular work.

The concave mirror is slightly tilted, so as to bring the focal plane a little to one side of the plane mirror, thus forming a Herschelian reflector: with the long focal ratio this does not materially affect definition, and it would be quite possible to alter the form to the Newtonian if found necessary. For stellar work the following is done by means of an eyepiece with double cross-wires near the edge of the plate, and the plate-holder and screw motions can be rotated to bring the star trails parallel with the R.A. screw motion.

For the Moon and planets a negative enlarging lens is employed, and it has been found that with suitable air-conditions an equivalent focal length of 120 feet can be employed with advantage. Although the resulting photograph is not usually so crisply defined as one taken in the primary focus, there is an evident gain in detail, and the disadvantage of "grain" in the plate is not so obtrusive.

In order to eliminate as far as possible uneven motion in the instrument, it is intended, when the enlarging lens is used, to move the plate-holder by means of an oil cylinder, instead of driving the cœlostat. By this means a smooth, continuous motion is possible, and there is the additional advantage that the concave mirror need not be moved in azimuth for the different declinations required, as it is not necessary for the plane mirror to be parallel with the polar axis. The instrument is also well adapted for large-scale photographs of the solar surface.

It has been found desirable to cover in the space between the cœlostat house and the concave mirror with a movable wooden tube; unless this is used, the ground air-currents interfere with good definition.

On Dr. Roberts' method of determining the absolute dimensions of an Algol Variable Star. By the Rev. J. Stein, S.J., Sc.D.

(*Communicated by Professor H. H. Turner.*)

In an interesting article of the January number, 1906, of the *M.N.* (vol. lxvi. p. 123), Dr. Alex. Roberts starts from the hypothesis that the light-changes of the Algol variables U Pegasi and RR Centauri are fairly well explained by two eclipsing stars moving in a circular orbit, and tries to reduce the residuals (O—C) by applying some corrections to the theoretical light-curve. He supposes that the orbit is passing through the line of sight, and puts—

P = Period of variable in minutes.

ϵ (very small) = Eccentricity of Orbit.

λ = Longitude of periastron, reckoned from line of sight.

θ = True angular distance of eclipsing star from line of sight at time T.

Δm = Change in magnitude per minute at time T.

l = Time (in minutes) that light takes to cross the semi-orbit.

Δt = Correction to the assumed time of passing the principal minimum phase.

$p = \epsilon \sin \lambda$, $q = \epsilon \cos \lambda$.

Then, neglecting the second and higher powers of ϵ, he proceeds from the typical equation of condition—

$$\Delta m \cos \theta \frac{2}{n} p + \Delta m \sin \theta \frac{2}{n} q + \Delta m \, \Delta t + \Delta m \, |\cos \theta| \, l + \ldots = (\text{O—C}),$$

where p, q, Δt, l are unknown quantities, $n = \frac{2\pi}{P}$ and $|\cos \theta|$ denotes the absolute value of $\cos \theta$.

According to the author, the three first terms are corrections depending on eccentricity and variation of epoch; the fourth term is the correction due to the fact that light takes an appreciable interval of time to traverse the orbit. (The other corrections will here be left out of consideration.) Hence, resolving the equations of condition after the method of least squares, Dr. Roberts finds l, and consequently the absolute dimensions of the orbit.

It seems, however, that the point of departure of this method is open to criticism. Indeed, the first member of the fundamental equation is not the adequate expression of the corrections due to eccentricity, variation of epoch, and aberration of light.

To show this, let us take the most simple case—that the two components are both spherical, and that the nearly circular relative orbit is passing through the line of sight. In this case the apparent brightness of the binary system is only depending on the apparent distance ρ of the centres of the two spheres, as seen from our Sun.

Hence we may put—

Change in magnitude in theoretical light-curve deduced from circular orbit,

$$\Delta m = \frac{dm}{d\rho}\frac{d\rho}{dt} \quad . \quad . \quad . \quad . \quad (1)$$

For the purpose of determining the effect of eccentricity on ρ, we may neglect the aberration of light. Thus, in the circular orbit we have—

$$\rho^2 = r^2 \sin^2\theta \quad . \quad . \quad . \quad . \quad (2)$$

r denoting the constant radius-vector, and θ referring to the star S_1 eclipsing at principal minimum.

Consequently, from (1) and (2),

$$\frac{dm}{d\rho} = \Delta m \tan\theta \frac{1}{n\rho} \quad . \quad . \quad . \quad . \quad (3)$$

Putting $\Delta\rho$, Δr, $\Delta\theta$ = the variations of ρ, r, θ, at the time T by passing from the circular orbit to the improved, eccentrical one, we find—

$$\frac{\Delta\rho}{\rho} = \frac{\Delta r}{r} + \cot\theta\,\Delta\theta \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{\Delta r}{r} = -\epsilon\cos(\theta-\lambda) = -q\cos\theta - p\sin\theta \quad . \quad . \quad (5)$$

$$\Delta\theta = 2\epsilon\sin(\theta-\lambda) + \text{const.}$$

For $\theta = 0$ we shall have $(\Delta\theta)_0 = n\,\Delta t$; consequently

$$\Delta\theta = 2\epsilon\sin(\theta-\lambda) + 2\epsilon\sin\lambda + n\,\Delta t = 2q\sin\theta + 2p(1-\cos\theta) + n\,\Delta t \quad (6)$$

Substituting (5) and (6) in (4),

$$\frac{\Delta\rho}{\rho} = q\cos\theta - p\frac{(1-\cos\theta)^2}{\sin\theta} + n\cot\theta\,\Delta t \quad . \quad . \quad (7)$$

Finally, according to (3) and (7), the adequate variation of magnitude at the time T, due to eccentricity and variation of epoch, will be

$$\frac{dm}{d\rho}\Delta\rho = \Delta m\sin\theta\frac{1}{n}q - \Delta m\frac{(1-\cos\theta)^2}{\cos\theta}\frac{1}{n}p + \Delta m\,\Delta t \quad (8)$$

Vice versa, for determining the effect of aberration, we may neglect the eccentricity of the orbit, and put—

a_1, a_2 = the semi-axes of the circular orbits of the two components S_1 and S_2 around the common centre of gravity.

ρ_1, ρ_2 = the apparent projections of the radii vectores on the tangential plane of the celestial sphere at the time T, this being reckoned from the moment of principal minimum.

l_1, l_2, σ = Time (in minutes) that light takes to cross the semi-

axes of the orbits and the distance from the centre of gravity to the Sun.

Then the time taken by the light—

to pass from S_1 to the Sun will be $= \sigma - l_1 \cos n(T-\sigma)$

„ „ S_2 „ „ $= \sigma + l_2 \cos n(T-\sigma)$

Consequently, putting $n(T-\sigma)=\theta$:

$$\rho_1 = a_1 |\sin(\theta + nl_1 \cos\theta)|$$
$$\rho_2 = a_2 |\sin(\theta - nl_2 \cos\theta)|$$

where $\cos\theta$ is to be taken with its proper sign.

The quantities l_1 and l_2 being generally very slight, and since $a_1 l_2 = a_2 l_1$, we may put—

$$\rho = \rho_1 + \rho_2 = a|\sin\{\theta + n(l_1 - l_2)\cos\theta\}| \quad . \quad (9)$$

and consequently the correction for light-equation will be

$$\Delta m(l_1 - l_2)\cos\theta \quad . \quad . \quad . \quad (10)$$

Thus it would seem theoretically impossible to determine the absolute dimensions of the orbit in this manner. If this statement is correct, it follows that the numerical results of Dr. Roberts' paper cannot have the interpretation which is given to them.

Specola Vaticana: 1908 *March* 19.

Note to Father Stein's paper. By Professor H. H. Turner, D.Sc., F.R.S.

THE following elementary geometrical considerations indicate the reasons why the light-equation fails to give us expected information about the orbit. They were mentioned at the meeting of the Society in communicating Father Stein's paper, and are added here as a note at the request of one of the Secretaries.

Consider the three cases—(*a*) when the mass of the bright body is so small as to be negligible; (*b*) when the mass of the dark body is negligible; (*c*) when the two bodies are of equal mass. And in each case let us suppose the orbit circular and in a plane passing through the line of sight, and that light takes a time 2T to cross the orbit.

(*a*) In the first case (fig. 1) the dark body is stationary at the centre O, and the bright body revolves round it, and is eclipsed at A, the Earth being in the direction of the arrow. The light which suffers eclipse reaches O at a time T *after* the actual eclipse.

(*b*) In the second case (fig. 2) the bright body is stationary at the centre O, and the dark body revolves round it and eclipses it when at B. But the light which suffers eclipse must leave O at a time T *before* the actual eclipse.

Hence, referring events to the time at which light leaves O (or more generally the line XOY), we see that in case (*a*) the apparent eclipse is *retarded* and in case (*b*) it is *accelerated* by an equal amount. What happens in case (*c*) when the bodies are equal?

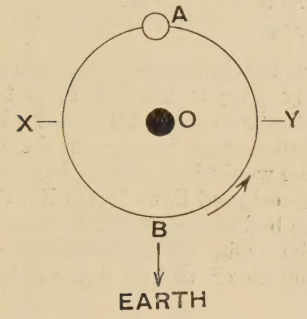


Fig. 1.

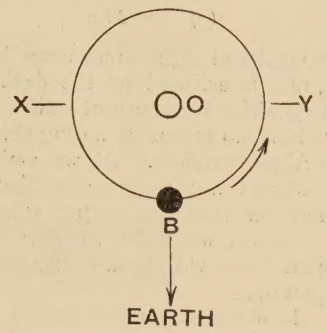


Fig. 2.

It is suggested that there is neither retardation nor acceleration, *i.e.* that the apparent eclipse takes place at the same time as the true eclipse, and this is actually the case. It was Professor Dyson who called attention to this case after reading Father Stein's paper at my request.

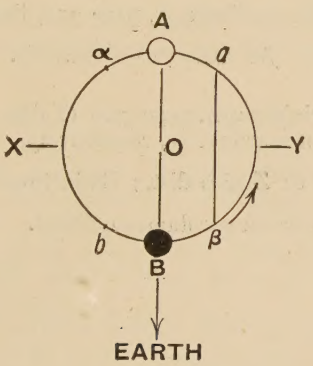


Fig. 3.

(*c*) When the masses are equal, they both revolve round O, their common C.G. The position for true eclipse is, bright body at A and dark at B; but the light which then leaves the bright body does not suffer eclipse, since when it reaches B the dark body will have moved away. Let aA, $A\alpha$, bB, $B\beta$ be four equal

small arcs described by either body in time T which light takes to cross the semi-orbit. Then the sequence of events is as follows:

	Bright Body.	Dark Body.	Eclipsed Light.
(1)	At a	At b	At a
(2)	At A	At B	In XOY
(3)	At a	At β	At β.

The eclipsed light must start before the bright body has reached A, and is eclipsed by the dark body after it has left B, so that the *position* for apparent eclipse is displaced from AB to $a\beta$. But we have no means of noting this displacement in space in the case of Algol variables. All we could note would be a change in epoch, and this is zero; for we see that the eclipsed light leaves XOY at the moment of true eclipse, when the bodies are at AB.

Hence, whatever the size of the orbit, when the masses are equal there will be no change in epoch of eclipse due to light-equation.

In other cases there will be a change of epoch,—a retardation if the dark body is larger, an acceleration if it is smaller. But the change will be small unless the masses are very unequal; and it will depend not only upon the size of the orbit, but on the ratio of the masses, which cannot be separately determined.

Errata.

In Professor Barnard's paper, page 357, line 27,

for Struve *read* Struve's.

In Mr. Baldwin's paper, page 369 (Table I.), the headings of the last four columns should be, respectively,

$d \log \sin^2 \mathrm{I}$; Zenith dist.; Extinction corr.; $\Delta \mathrm{M}$;

and the bracket over these columns omitted.